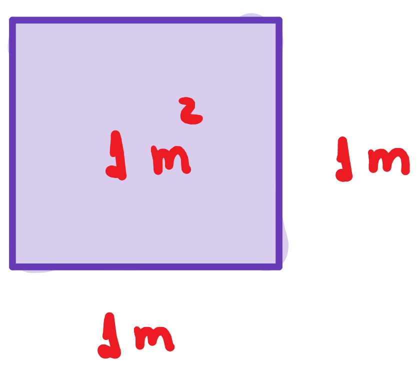
ÁREAS DE POLÍGONOS

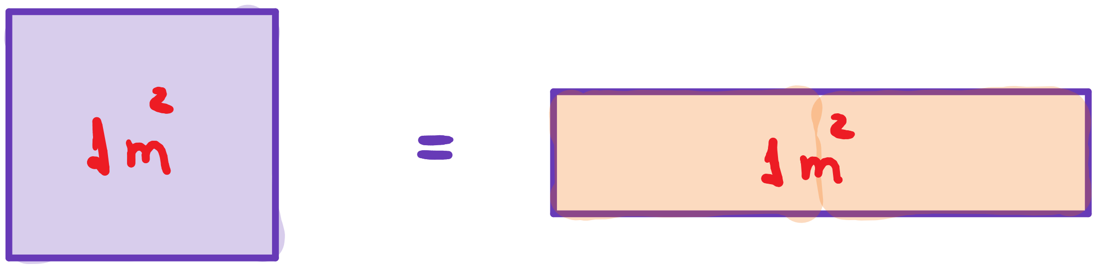
NOÇÃO DE ÁREA

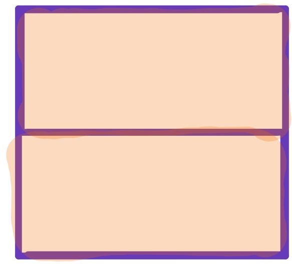
ESPAÇO OCUPADA NO PLANO.

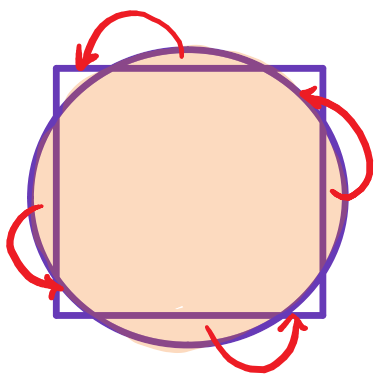
$1m^2$



MAS O FORMATO NÃO PRECISA SER QUADRADO!







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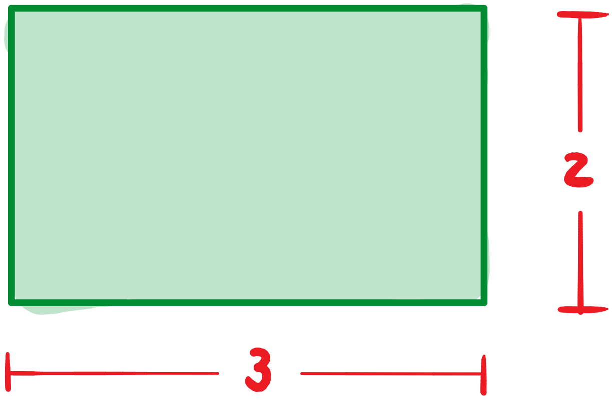
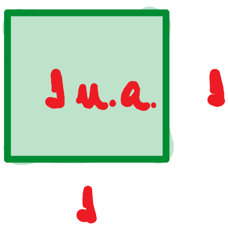
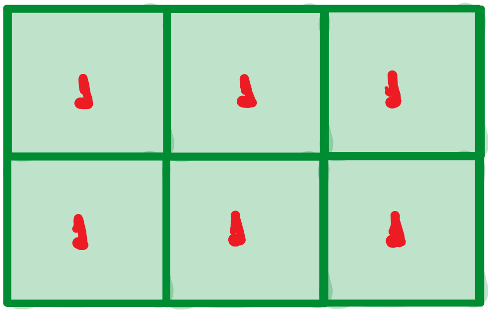
ÁREA DO RETÂNGULO

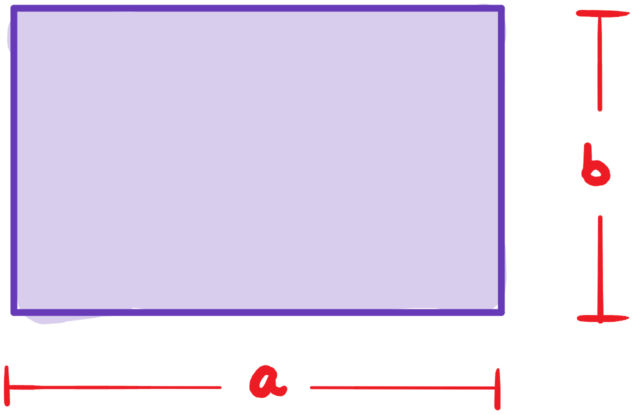
SUBDIVISÃO EM QUADRADOS UNITÁRIOS.

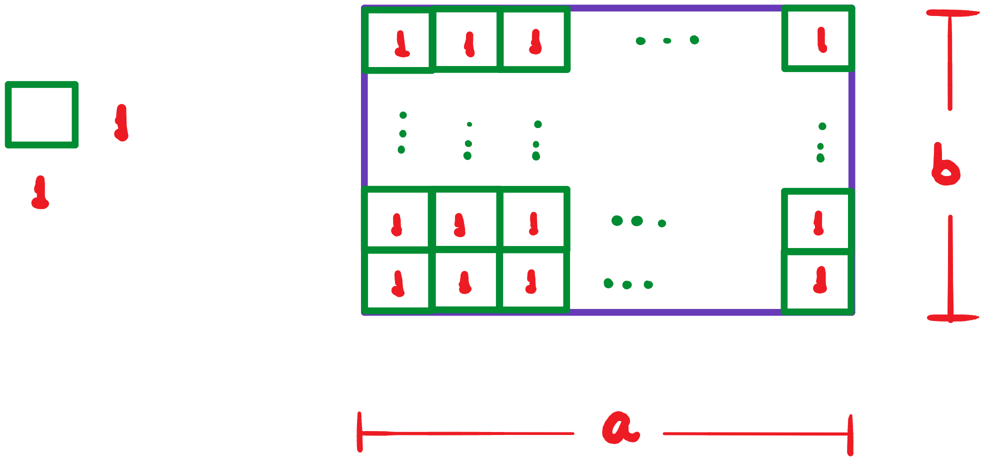






$A = 6$ u.a.



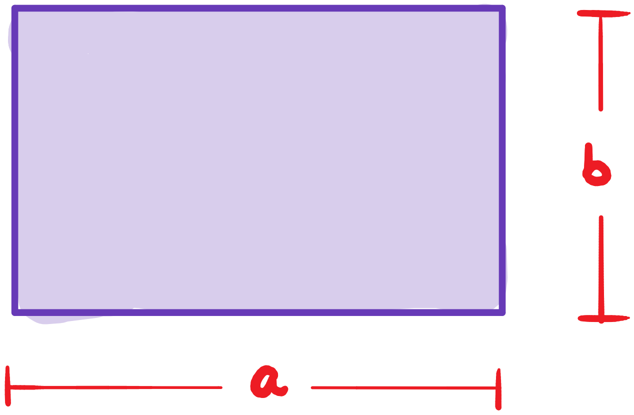


QUADRADOS NO COMPRIMENTO: a

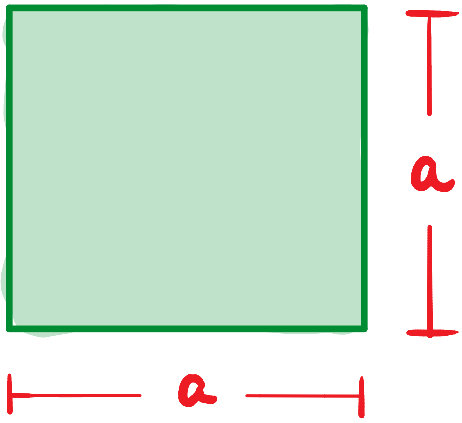
QUADRADOS NA LARGURA: b

TOTAL DE QUADRADOS: $a.b$

ÁREA: $\boxed{A = a.b}$

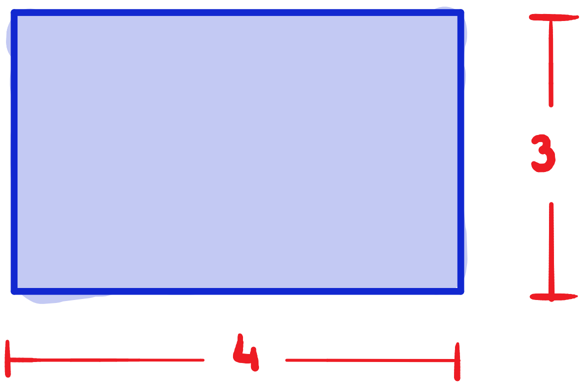


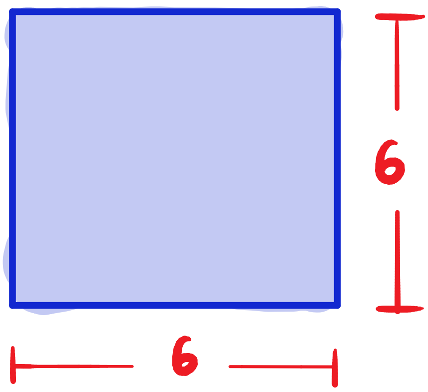
$$\underline{A = a \cdot b}$$



$$A = a \cdot a$$

$$\underline{A = a^2}$$





$$A = 4 \cdot 3$$

$$A = 12 \text{ u.a.}$$

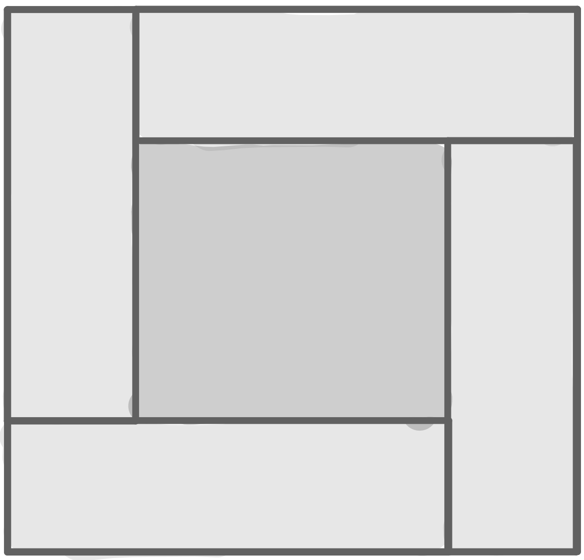
$$A = 6^2$$

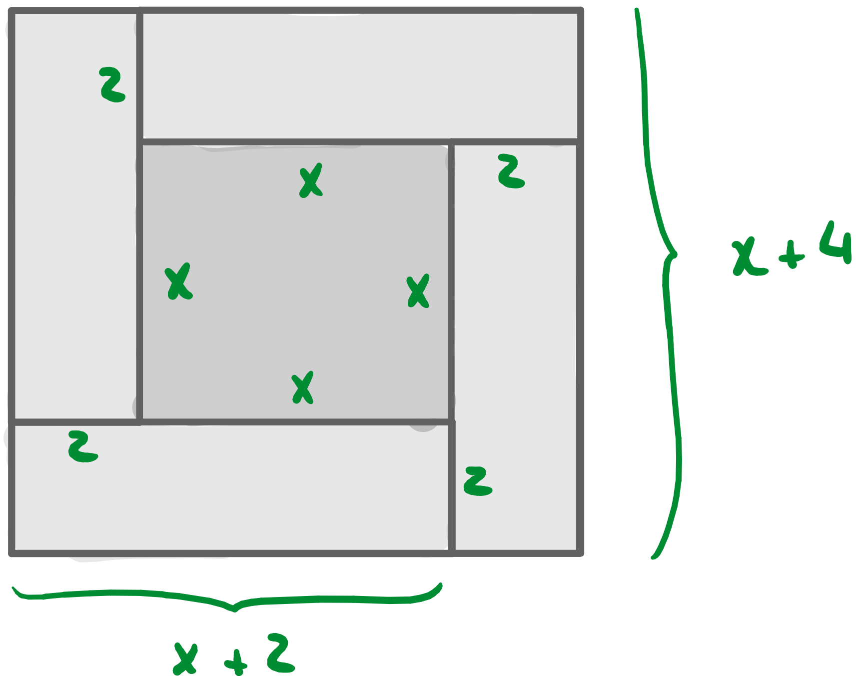
$$A = 36 \text{ u.a.}$$

EXEMPLO

UMA SALA QUADRADA SERÁ DIVIDIDA EM UM QUADRADO E QUATRO RETÂNGULOS IGUAIS, CONFORME A FIGURA.

SE A MENOR DIMENSÃO DOS RETÂNGULOS É IGUAL A 2 METROS E A SOMA DAS ÁREAS DOS RETÂNGU-LOS É O TRIPLO DA ÁREA DO QUADRADO CENTRAL, QUAL A MEDIDA DO LADO DESSE QUADRADO?





$$4 \cdot 2(x+2) = 3 \cdot x^2$$

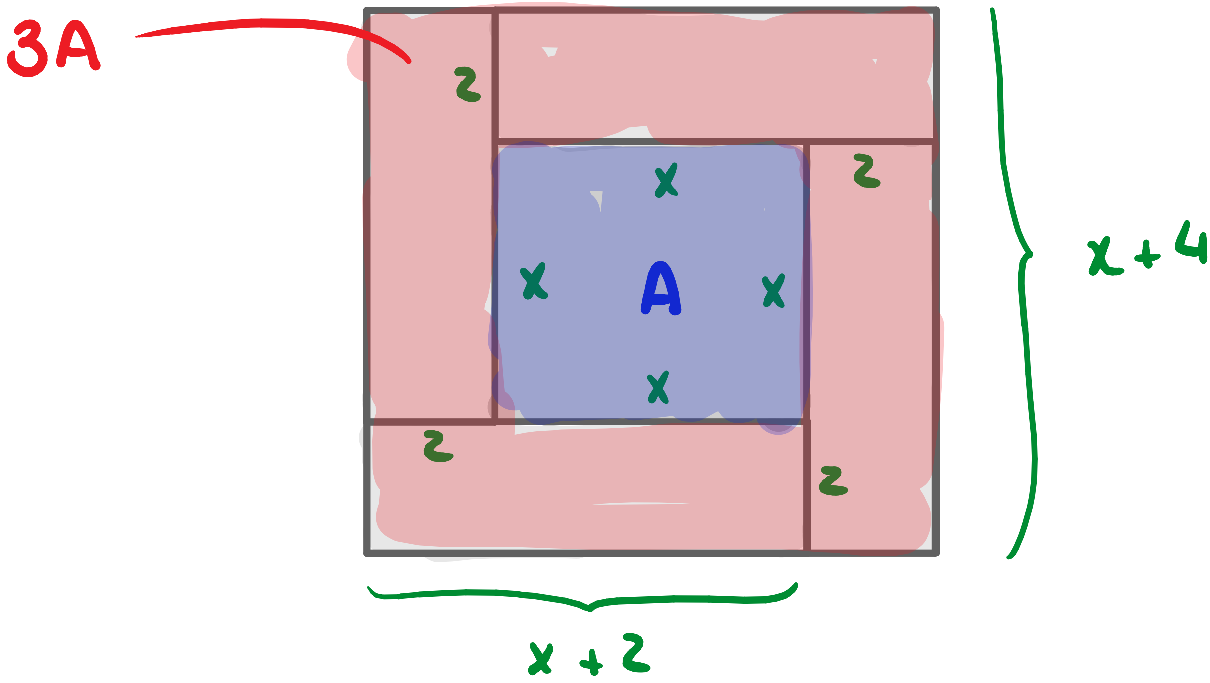
$$3x^2 - 8x - 16 = 0$$

$$\Delta = (-8)^2 - 4 \cdot 3 \cdot (-16) = 64 + 192 = 256$$

$$x = \frac{8 \pm \sqrt{256}}{2 \cdot 3}$$

$$x' = \frac{8 + 16}{6} \qquad x'' = \frac{8 - 16}{6}$$

$$\underline{x' = 4} \qquad x'' = \frac{-4}{3}$$



$$A_G = 4 . A_P$$

$$(x+4)^2 = 4 . x^2$$

$$(x+4)^2 = (2x)^2$$

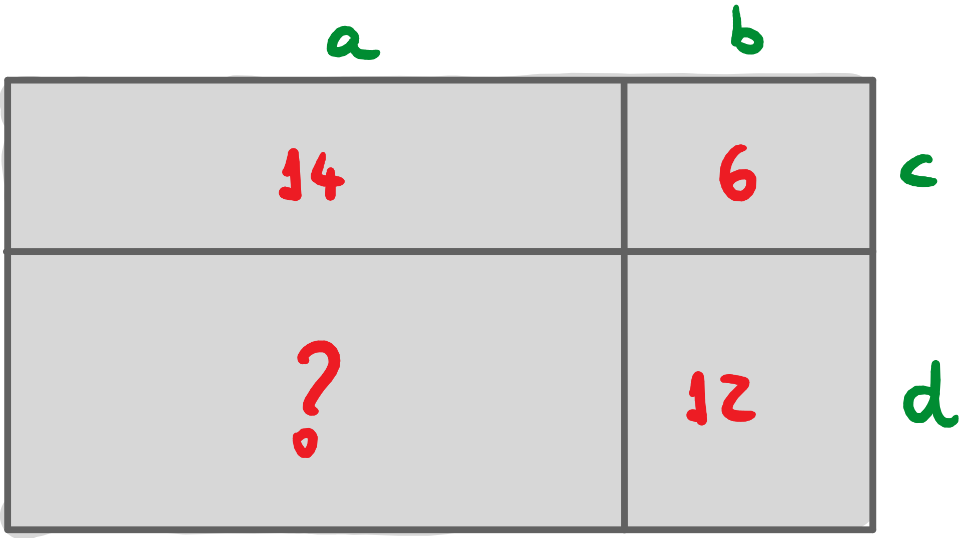
$$x + 4 = 2x \qquad \cancel{2x = -(x+4)}$$

$$\underline{x = 4}$$

EXEMPLO

UM RETÂNGULO FOI DIVIDIDO EM 4 RETÂNGULOS, CUJAS ÁREAS SÃO MOSTRADAS NA FIGURA.

CALCULE A ÁREA DO LOTE REMANESCENTE.



$a \cdot c = 14$ $\qquad$ $b \cdot d = 12$

$b \cdot c = 6$ $\qquad$ $A = a \cdot d$

$$\underbrace{a \cdot c}_{14} \cdot \underbrace{b \cdot d}_{12} = a \cdot d \cdot \underbrace{b \cdot c}_{6}$$

$$14 \cdot 12 = A \cdot 6$$

$$\cancel{6} A = 14 \cdot \cancelto{2}{12}$$

$$\underline{A = 28}$$

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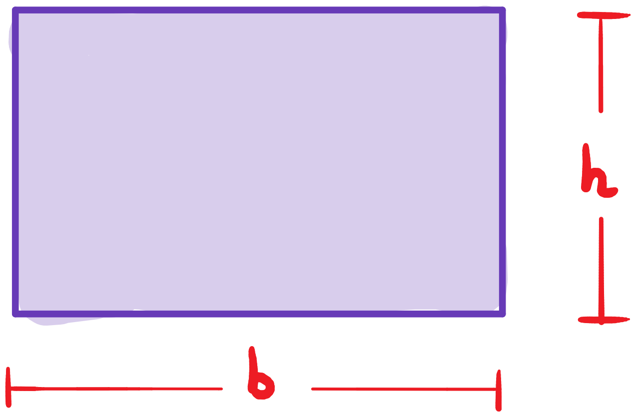
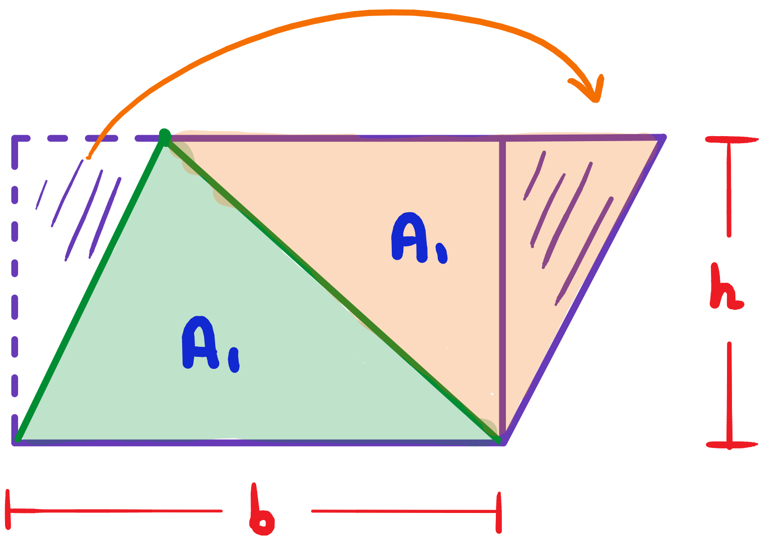
ÁREA DO TRIÂNGULO #1

A ÁREA MAIS IMPORTANTE, A PARTIR DA QUAL DERIVAREMOS TODAS AS OUTRAS.

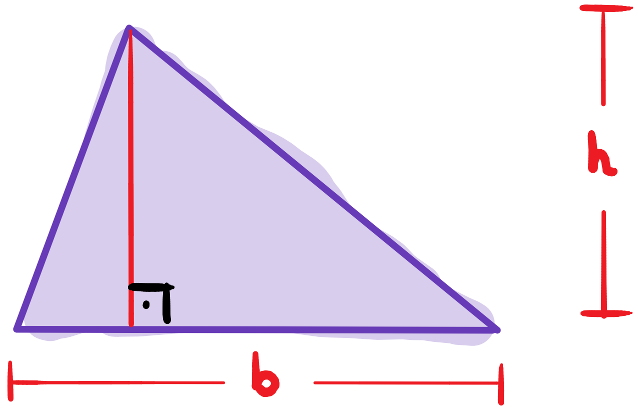


$$A = b \cdot h$$

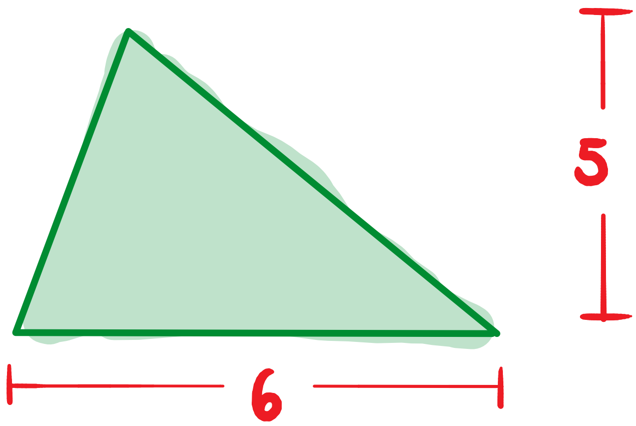


$$2 \cdot A_1 = b \cdot h$$

$$A_1 = \frac{b \cdot h}{2}$$



$$\boxed{A = \frac{1}{2} \cdot b \cdot h}$$



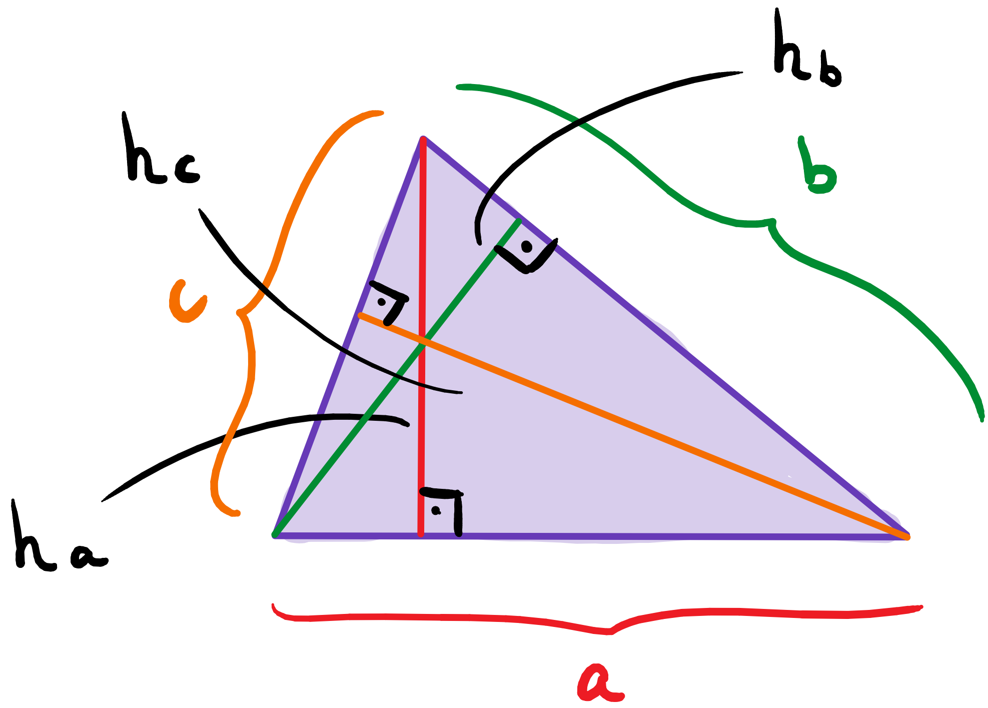
$$A = \frac{1}{\cancel{2}} \cdot \overset{3}{\cancel{6}} \cdot 5$$

$$A = 15$$

ALTURA DE TRIÂNGULO

ALTURA DO TRIÂNGULO É A DISTÂNCIA DO VÉRTICE AO LADO OPOSTO (OU AO PROLONGAMENTO DESTE).

TRIÂNGULO ACUTÂNGULO



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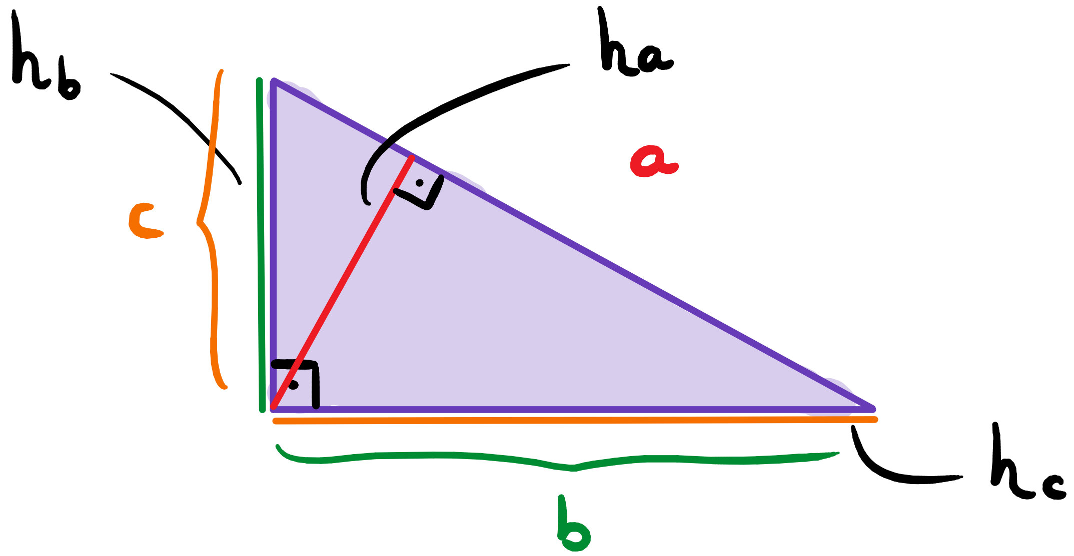
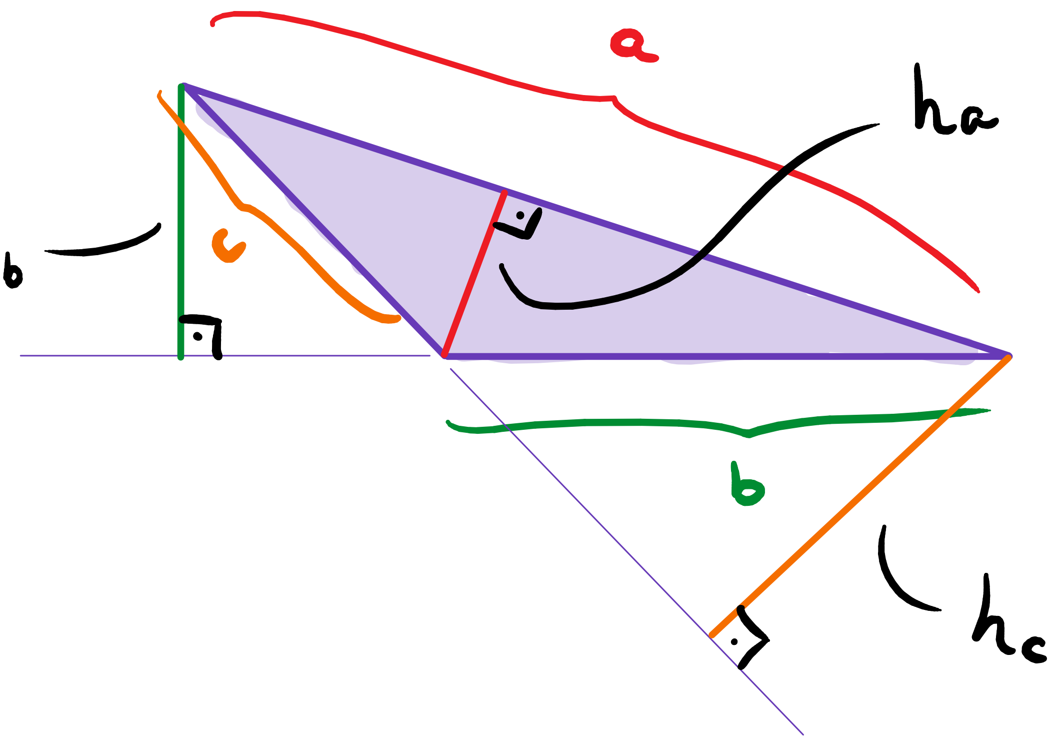
TRIÂNGULO RETÂNGULO



TRIÂNGULO OBTUSÂNGULO



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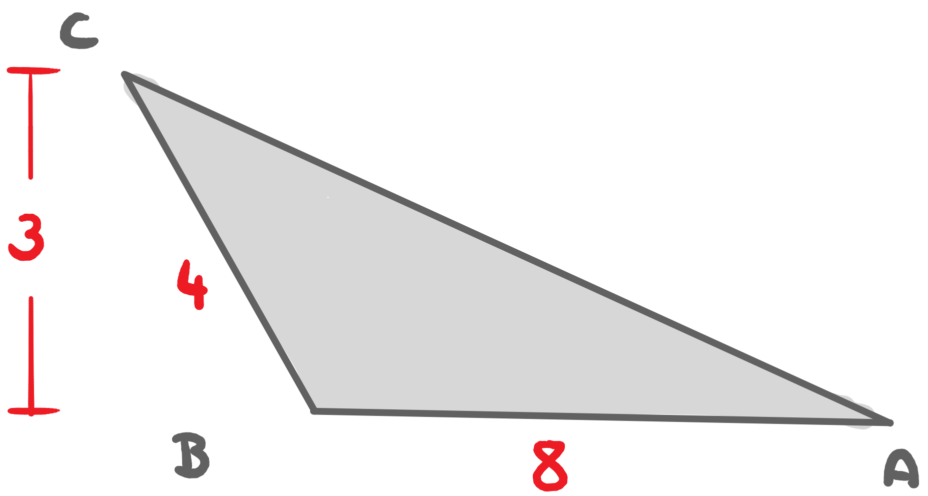
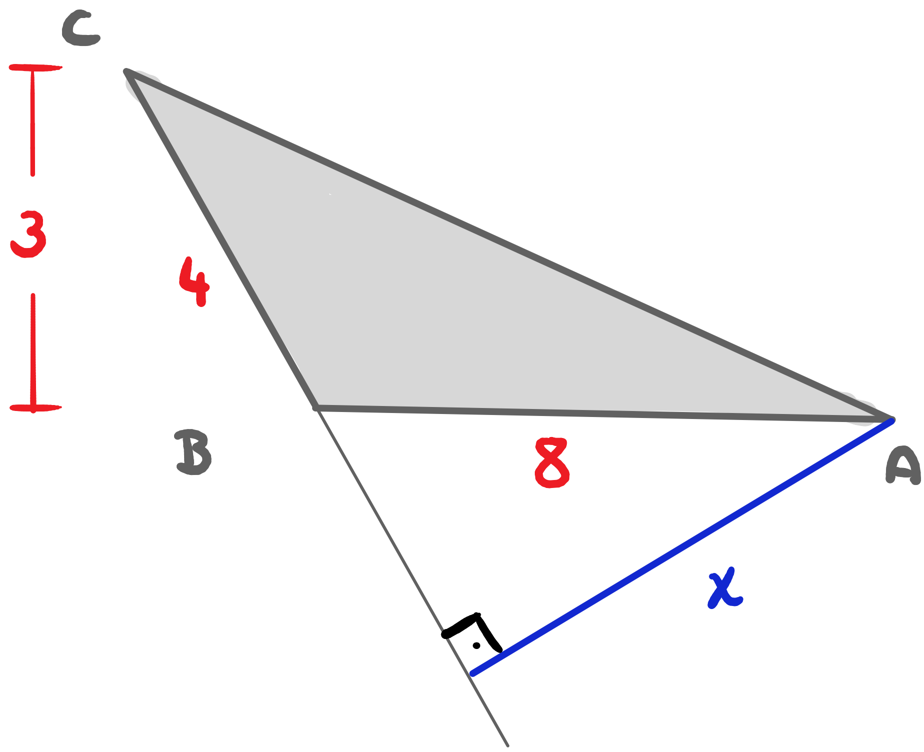
EXEMPLO

CALCULE A ALTURA RELATIVA AO LADO BC DO TRIÂNGULO ABAIXO.





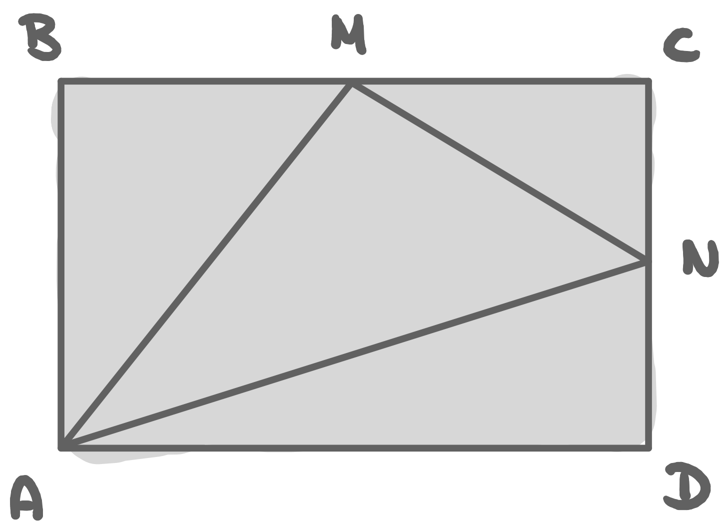
$$A_{\Delta} = A_{\Delta}$$

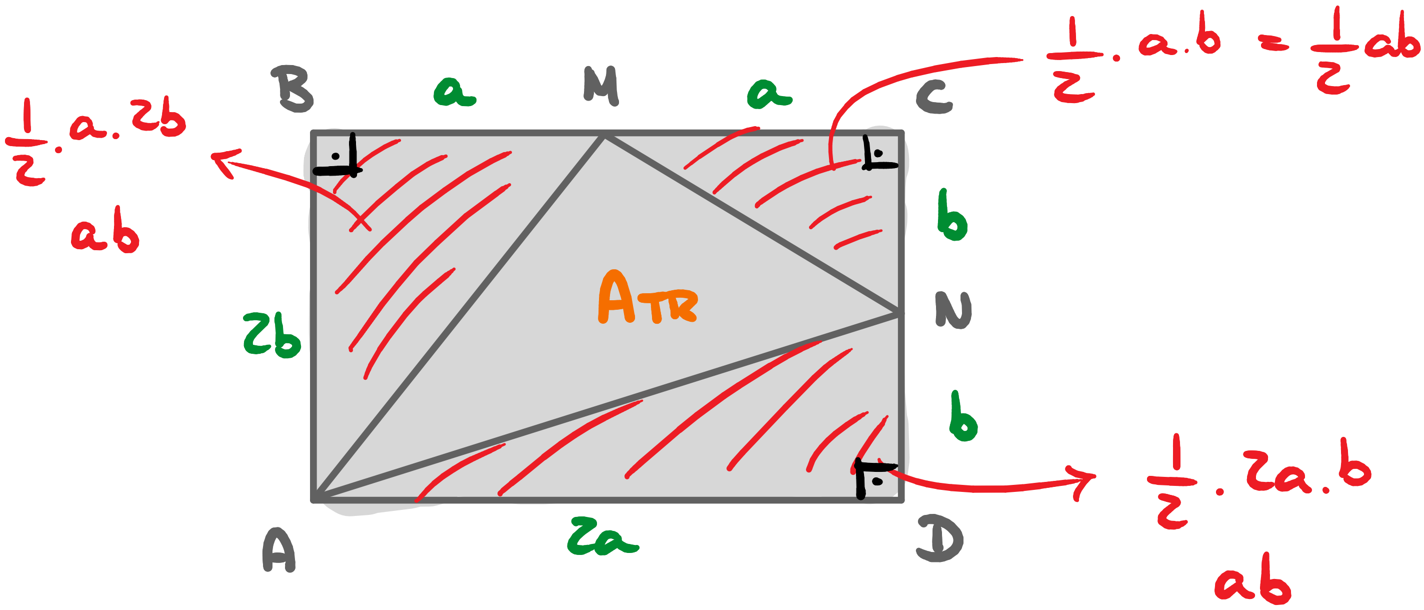
$$\frac{1}{\cancel{2}} \cdot \overset{2}{\cancel{8}} \cdot 3 = \frac{1}{\cancel{2}} \cdot \cancel{4} \cdot x$$

$$\underline{x = 6}$$

EXEMPLO

SABENDO QUE M E N SÃO PONTOS MÉDIOS DOS LADOS DO RETÂNGULO, CALCULE A RAZÃO ENTRE AS ÁREAS DO TRIÂNGULO AMN E DO RETÂNGULO ABCD.





$$A_{TOTAL} = 2a.2b$$

$$A_{TOTAL} = 4ab$$

$$A_{TR} = 4ab - \left(ab + \frac{1}{2}ab + ab\right)$$

$$A_{TR} = \frac{8ab}{2} - \frac{5}{2}ab$$

$$A_{TR} = \frac{3}{2}ab$$

$$R = \frac{\frac{3}{2}\cancel{ab}}{4\cancel{ab}} = \frac{3}{2} \cdot \frac{1}{4} \rightarrow R = \frac{3}{8}$$

EXEMPLO

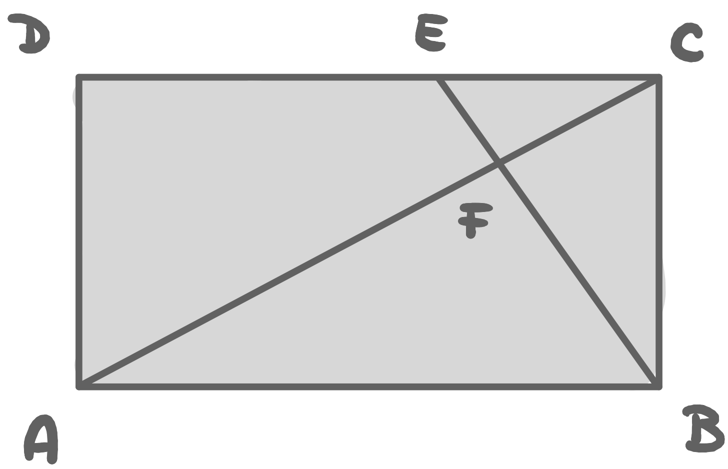
NO RETÂNGULO ABCD ABAIXO, TEM-SE:

AB = 6

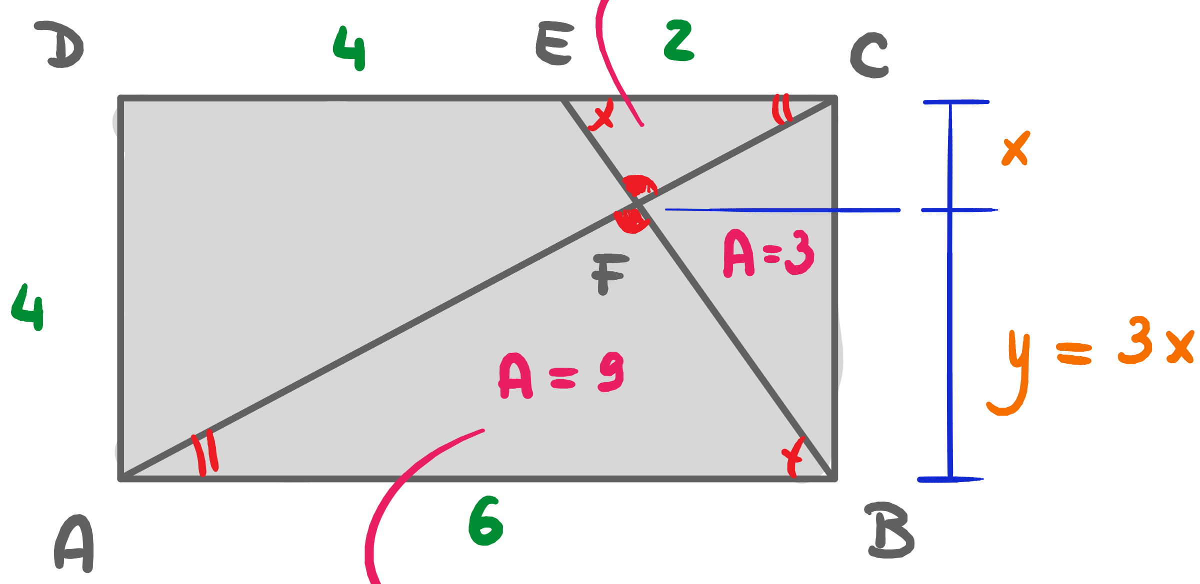
BC = 4

CE = 2

DETERMINE A ÁREA DO TRIÂNGULO BCF.



$$A = \frac{1}{2} \cdot 2 \cdot 1 = 1$$



$$A = \frac{1}{2} \cdot 6 \cdot 3 = 9$$

$$\Delta ABF \sim \Delta CEF \left(k = \frac{6}{2} = 3 \right)$$

$$3x + x = 4$$

$$\underline{x = 1}$$

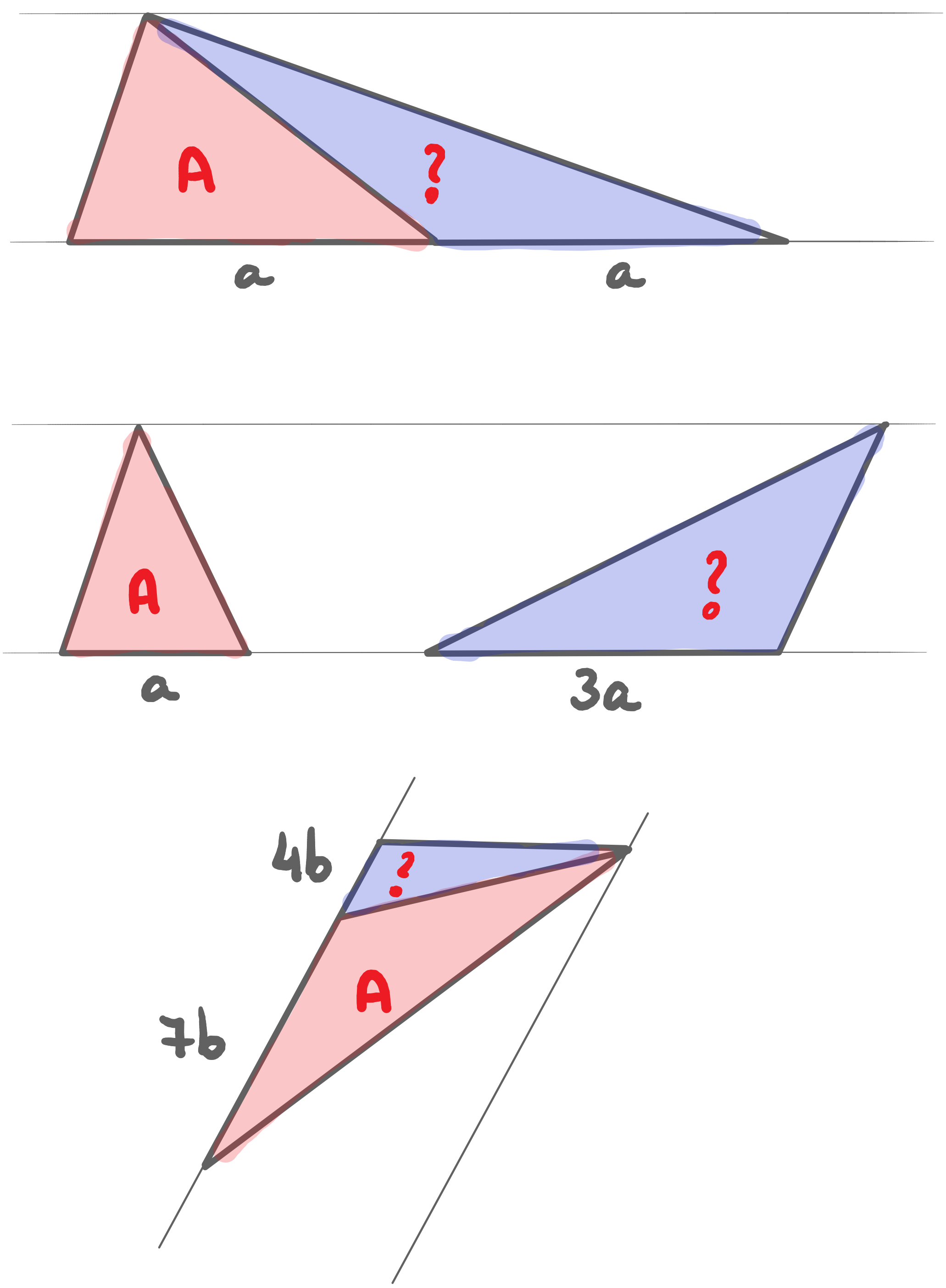
$$A_{ABC} = \frac{1}{2} \cdot 6 \cdot 4 = 12$$

$$A_{BCF} = \underset{12}{A_{ABC}} - \underset{9}{A_{ABF}}$$

$$\underline{A_{BCF} = 3}$$

EXEMPLO

DETERMINE A ÁREA DOS TRIÂNGULOS ABAIXO:



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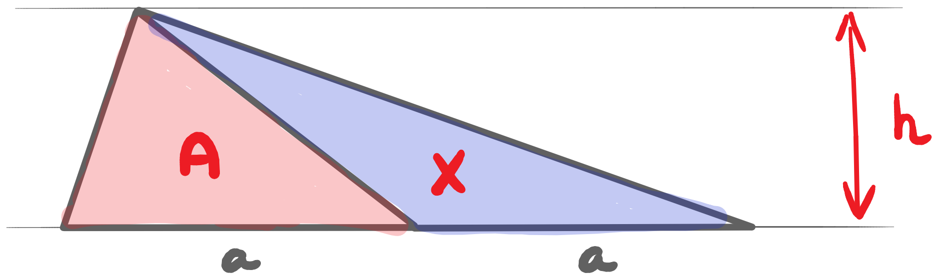
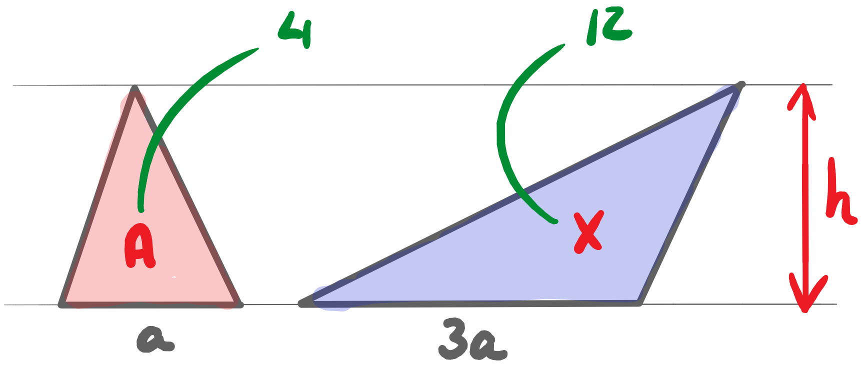
MESMA ALTURA → ÁREA PROPORCIONAL À BASE

MESMA BASE → ÁREA PROPORCIONAL À ALTURA



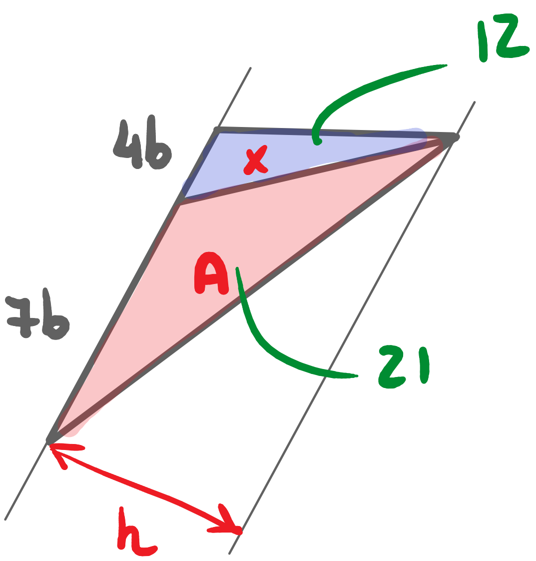
$$\frac{x}{\cancel{a}} = \frac{A}{\cancel{a}}$$

$$x = A$$



$$\frac{x}{A} = \frac{3\cancel{a}}{\cancel{a}}$$

$$x = 3A$$



$$\frac{x}{A} = \frac{4b}{7b}$$

$$x = \frac{4}{7}A$$

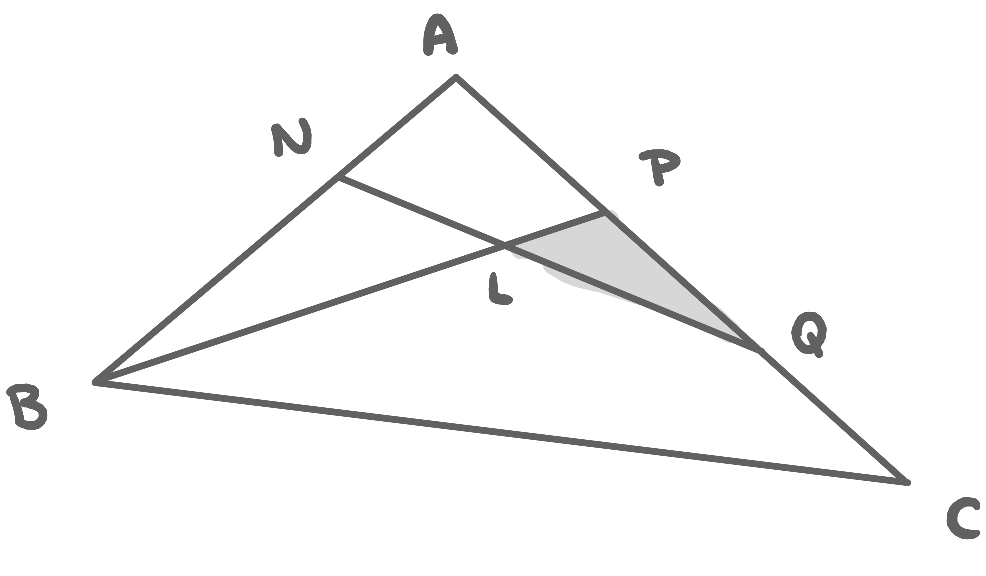
EXEMPLO

O TRIÂNGULO ABC ABAIXO POSSUI ÁREA 60.

SABENDO QUE:

$AP = PQ = QC$ E $NB = 2.NA$

CALCULE A ÁREA DO TRIÂNGULO PQL.



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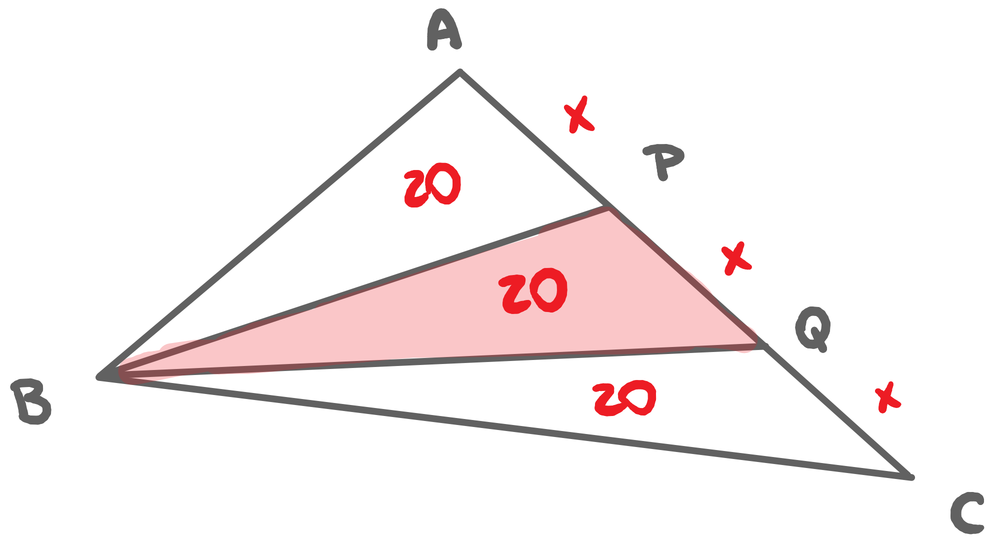

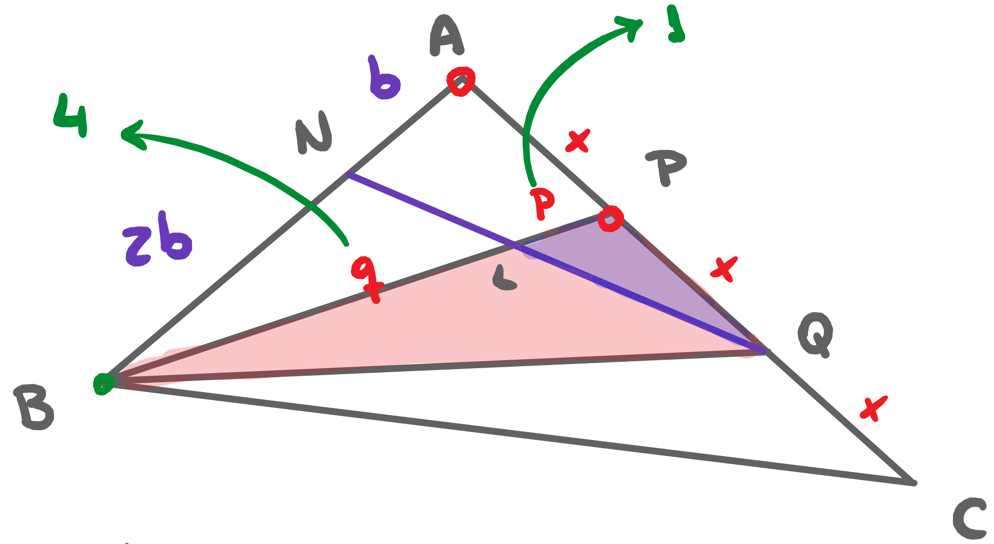

ΔAPB | RETA NQ :

$$\frac{q}{p} \cdot \frac{\cancel{x}}{2\cancel{x}} \cdot \frac{\cancel{b}}{2\cancel{b}} = 1 \longrightarrow \frac{q}{p} = 4$$

$$\frac{A_{PQL}}{1} = \frac{\cancel{A_{PQB}}^{\,20}}{5} \longrightarrow A_{PQL} = 4$$

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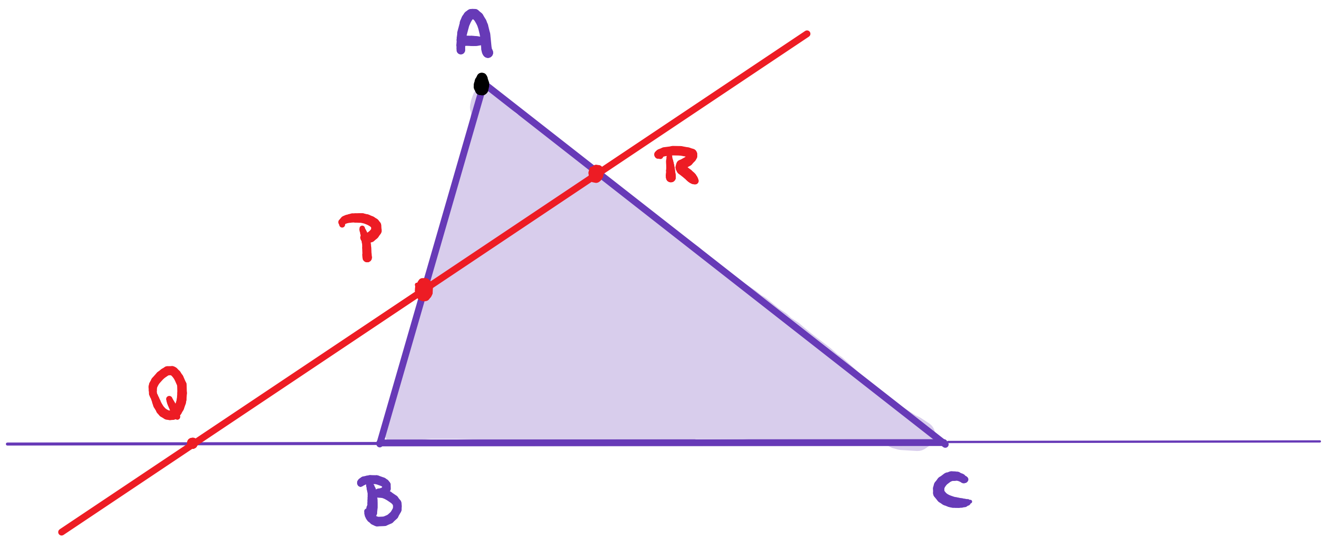
Teorema de Menelaus



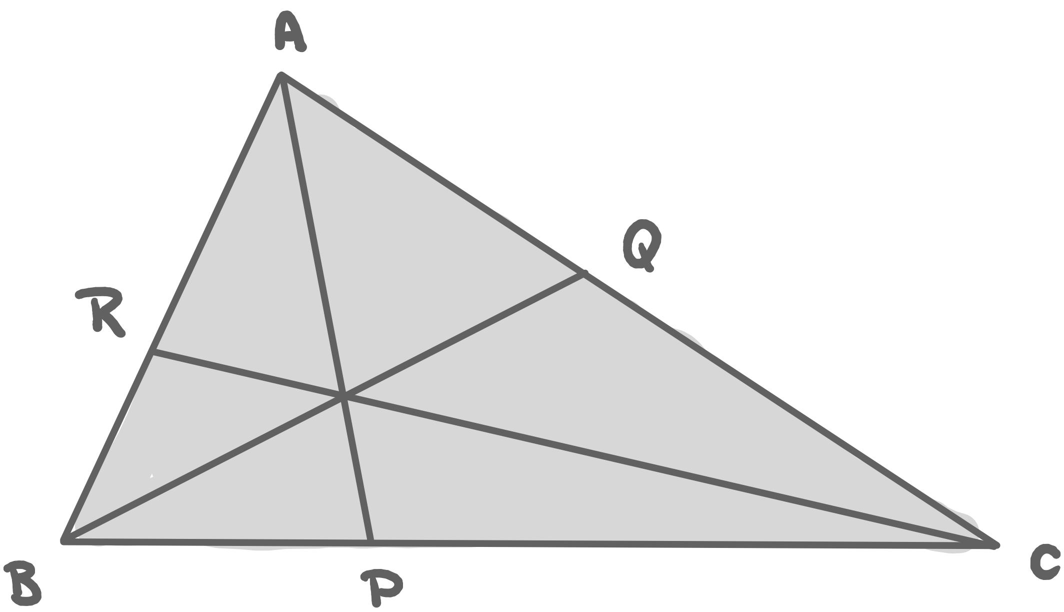
$$\frac{PA}{PB} \cdot \frac{QB}{QC} \cdot \frac{RC}{RA} = 1$$

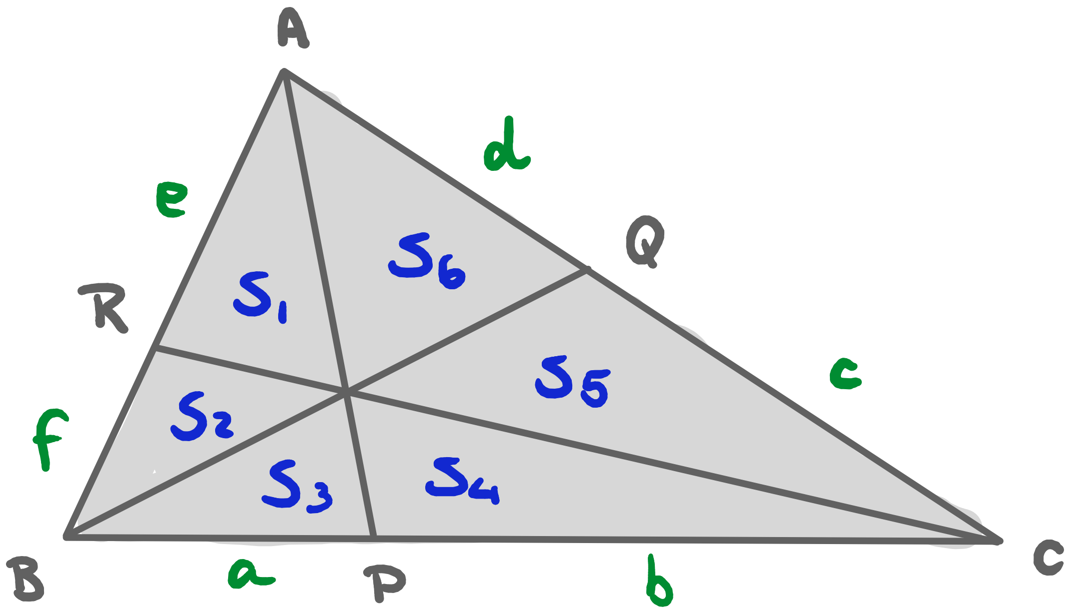
EXEMPLO

SEJA UM TRIÂNGULO ABC QUALQUER.
SEJAM TAMBÉM AS CEVIANAS AP, BQ E CR, CONCORRENTES EM UM ÚNICO PONTO.

DEMONSTRE O TEOREMA DE CEVA, OU SEJA:

$$\frac{RA}{RB} \cdot \frac{PB}{PC} \cdot \frac{QC}{QA} = 1$$





$$\frac{a}{b} = \frac{S_3}{S_4}$$

$$\frac{x}{y} = \frac{w}{z} = \frac{x+w}{y+z} = \frac{x-w}{y-z}$$

$$\frac{S_1 + S_2 + S_3}{S_4 + S_5 + S_6} = \frac{a}{b}$$

$$\frac{a}{b} = \frac{S_1 + S_2 + S_3}{S_4 + S_5 + S_6} = \frac{S_3}{S_4} = \frac{S_1 + S_2}{S_5 + S_6}$$

$$\frac{a}{b} = \frac{S_1 + S_2}{S_5 + S_6}$$

$$\frac{c}{d} = \frac{S_5}{S_6} = \frac{S_3 + S_4 + S_5}{S_1 + S_2 + S_6} = \frac{S_3 + S_4}{S_1 + S_2}$$

$$\frac{c}{d} = \frac{S_3 + S_4}{S_1 + S_2}$$

$$\frac{e}{f} = \frac{S_1}{S_2} = \frac{S_1 + S_5 + S_6}{S_2 + S_3 + S_4} = \frac{S_5 + S_6}{S_3 + S_4}$$

$$\frac{e}{f} = \frac{S_5 + S_6}{S_3 + S_4}$$

$$\frac{RA}{RB} \cdot \frac{PB}{PC} \cdot \frac{QC}{QA} = \frac{e}{f} \cdot \frac{a}{b} \cdot \frac{c}{d}$$

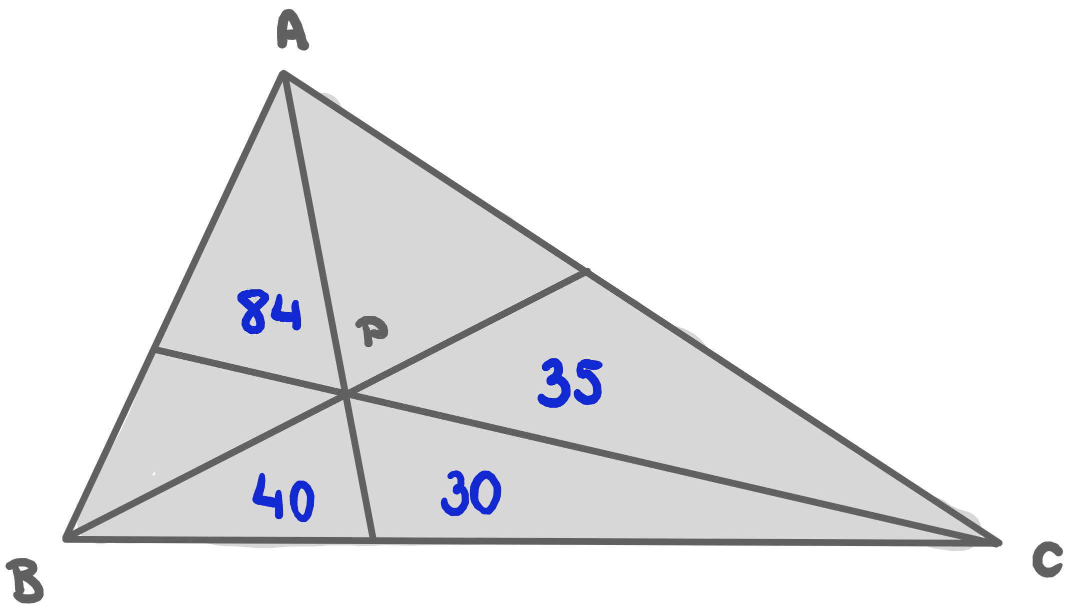
$$\frac{RA}{RB} \cdot \frac{PB}{PC} \cdot \frac{QC}{QA} = \frac{\cancel{S_5 + S_6}}{\cancel{S_3 + S_4}} \cdot \frac{\cancel{S_1 + S_2}}{\cancel{S_5 + S_6}} \cdot \frac{\cancel{S_3 + S_4}}{\cancel{S_1 + S_2}}$$

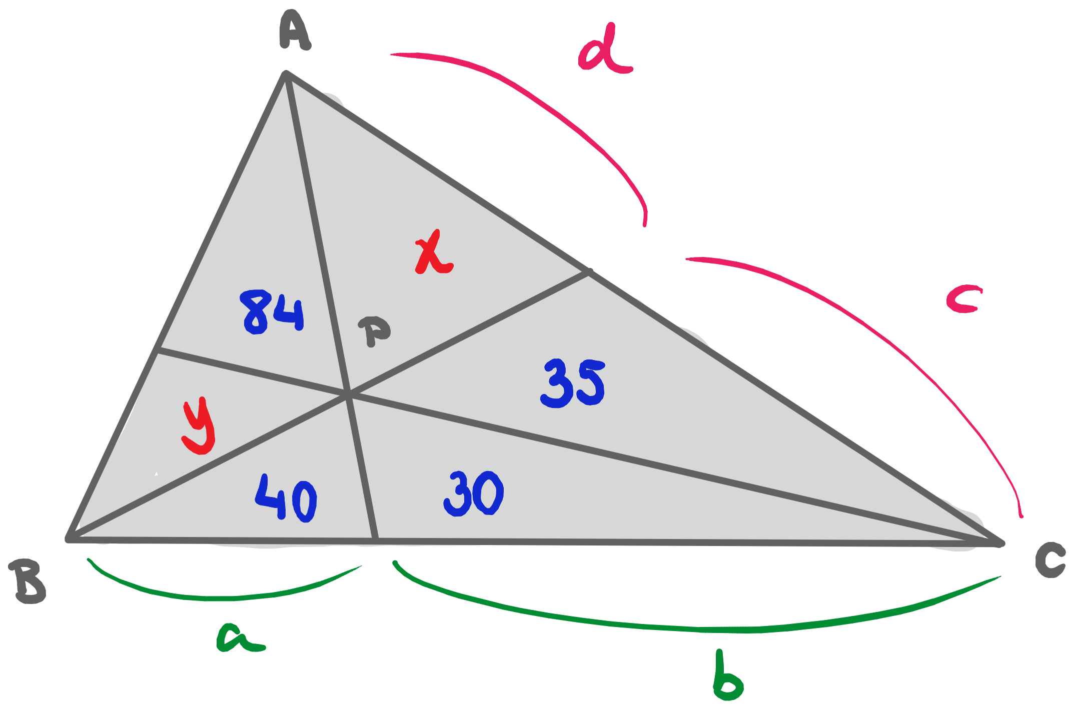
$$\boxed{\frac{RA}{RB} \cdot \frac{PB}{PC} \cdot \frac{QC}{QA} = 1}$$

EXEMPLO

SEJA P O PONTO INTERNO AO TRIÂNGULO ABC, DIVIDINDO-O EM 6 PARTES, COM ALGUMAS DAS ÁREAS MOSTRADAS.

CALCULE A ÁREA DO TRIÂNGULO ABC.





$$\cancel{\frac{a}{b}} = \boxed{\frac{4\cancel{0}}{3\cancel{0}} = \frac{y+84+40}{x+35+30}}$$

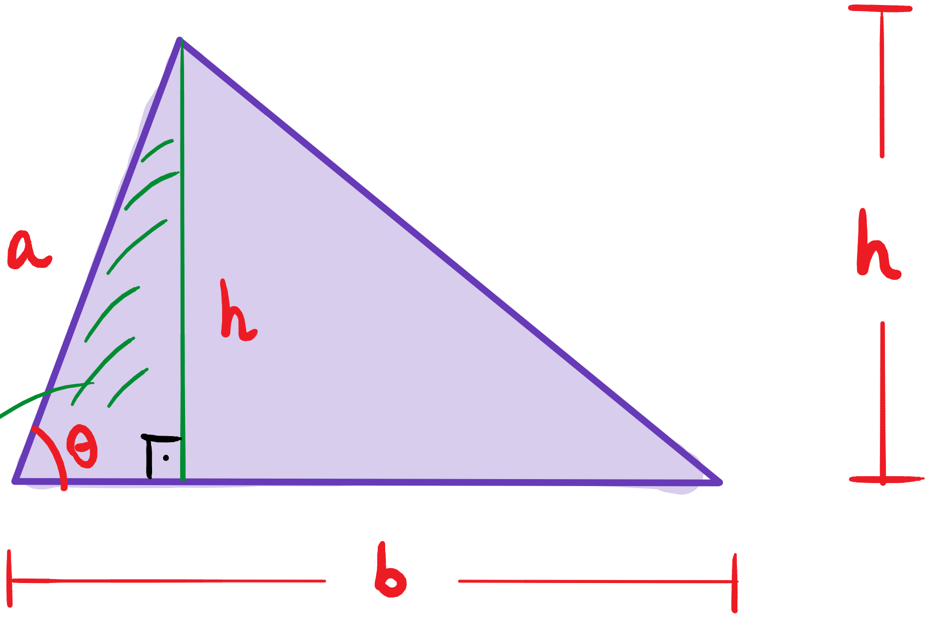
$$\cancel{\frac{d}{c}} = \boxed{\frac{x}{35} = \frac{x+y+84}{40+30+35}}$$

$$x = 70 \quad ; \quad y = 56$$

$$A_{ABC} = \overbrace{40+30}^{70} + \overbrace{84+56}^{140} + \overbrace{35+70}^{105}$$

$$\underline{A_{ABC} = 315}$$

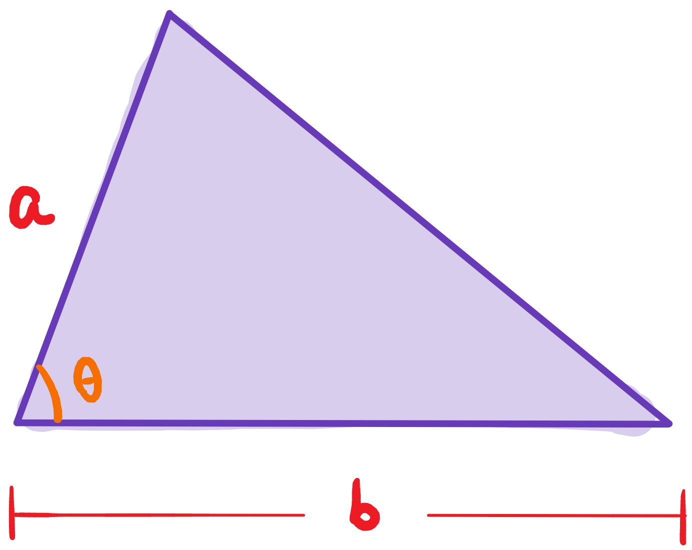
ÁREA DO TRIÂNGULO #2



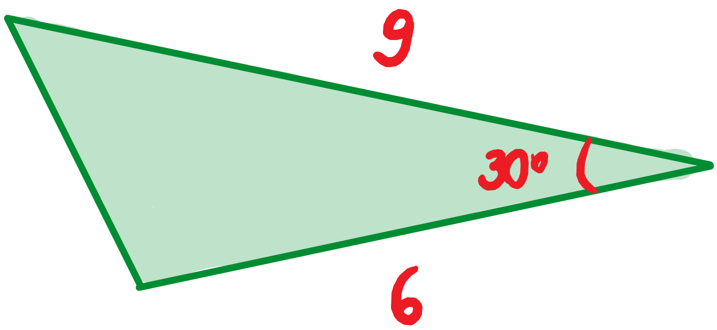
$$A = \frac{1}{2} b \cdot h$$

$$\operatorname{sen}\theta = \frac{h}{a} \rightarrow h = a \cdot \operatorname{sen}\theta$$

$$A = \frac{1}{2} \cdot b \cdot a \cdot \operatorname{sen}\theta$$



$$\boxed{A = \frac{1}{2}\, ab . \operatorname{sen}\theta}$$

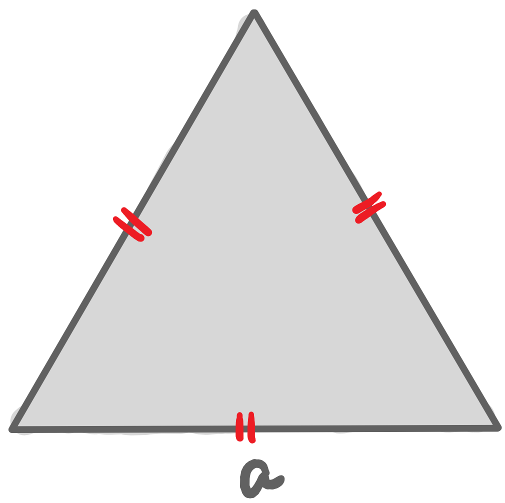


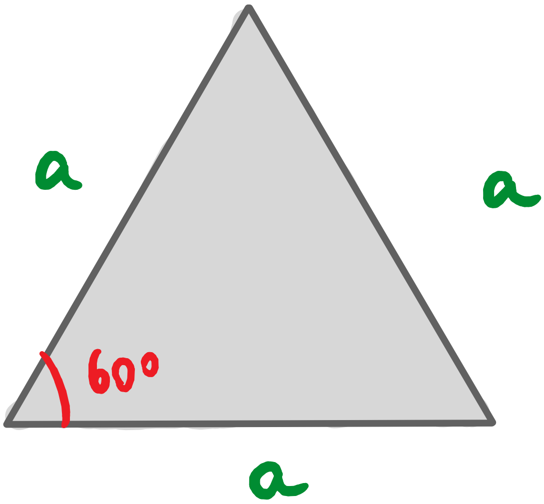
$$A = \frac{1}{\cancel{2}} . 9 . \overset{3}{\cancel{6}} . \operatorname{sen} 30^\circ$$

$$A = 9 . 3 . \frac{1}{2} \longrightarrow A = \frac{27}{2}$$

EXEMPLO

CALCULE A ÁREA DE UM TRIÂNGULO EQUILÁTERO DE LADO a.



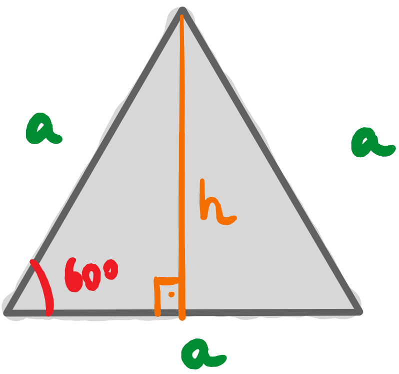


$$A_\Delta = \frac{1}{2} \cdot a \cdot b \cdot \operatorname{sen}\theta$$

$$A_\Delta = \frac{1}{2} \cdot a \cdot a \cdot \operatorname{sen} 60°$$

$$A_\Delta = \frac{1}{2} \cdot a^2 \cdot \frac{\sqrt{3}}{2} \rightarrow \boxed{A_{\Delta EQ} = \frac{a^2\sqrt{3}}{4}}$$

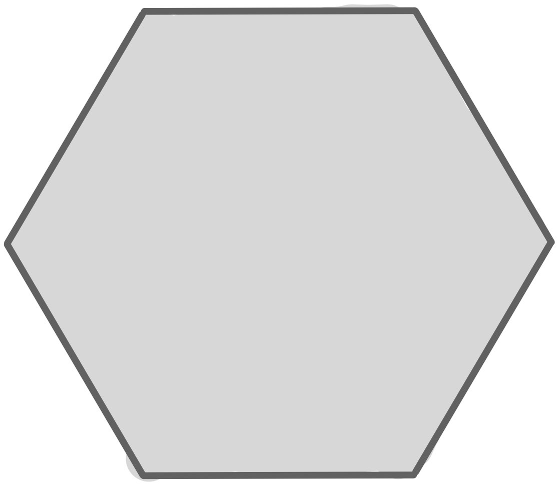
$$\operatorname{sen} 60° = \frac{h}{a} \rightarrow \frac{\sqrt{3}}{2} = \frac{h}{a} \rightarrow h = \frac{a\sqrt{3}}{2}$$

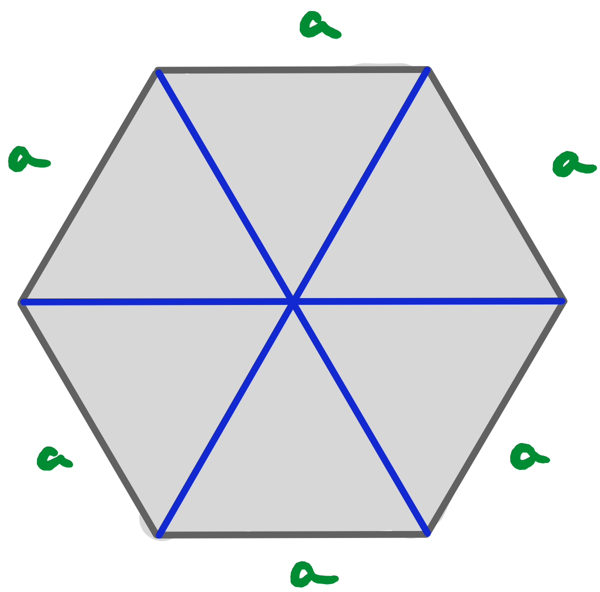


$$A = \frac{1}{2} \cdot a \cdot a \frac{\sqrt{3}}{2} = \frac{a^2\sqrt{3}}{4}$$

EXEMPLO

CALCULE A ÁREA DE UM HEXÁGONO REGULAR DE LADO a.





$$A_{HEX} = 6 . A_{EQ}$$

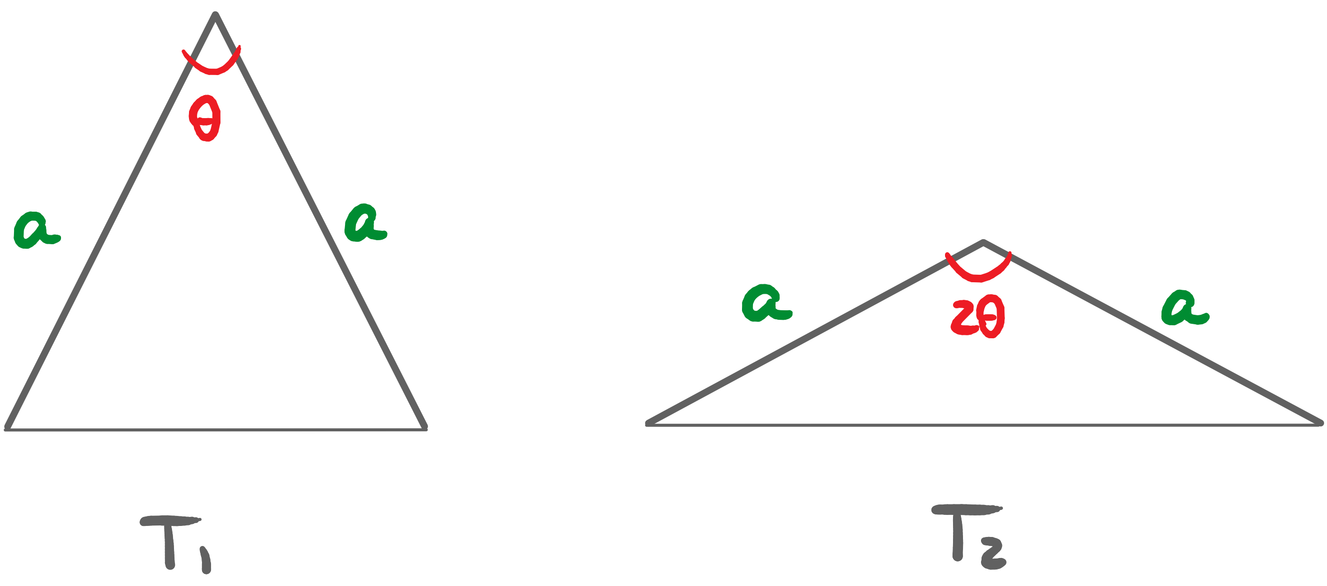
$$A_{HEX} = 6 . \frac{a^2\sqrt{3}}{4}$$

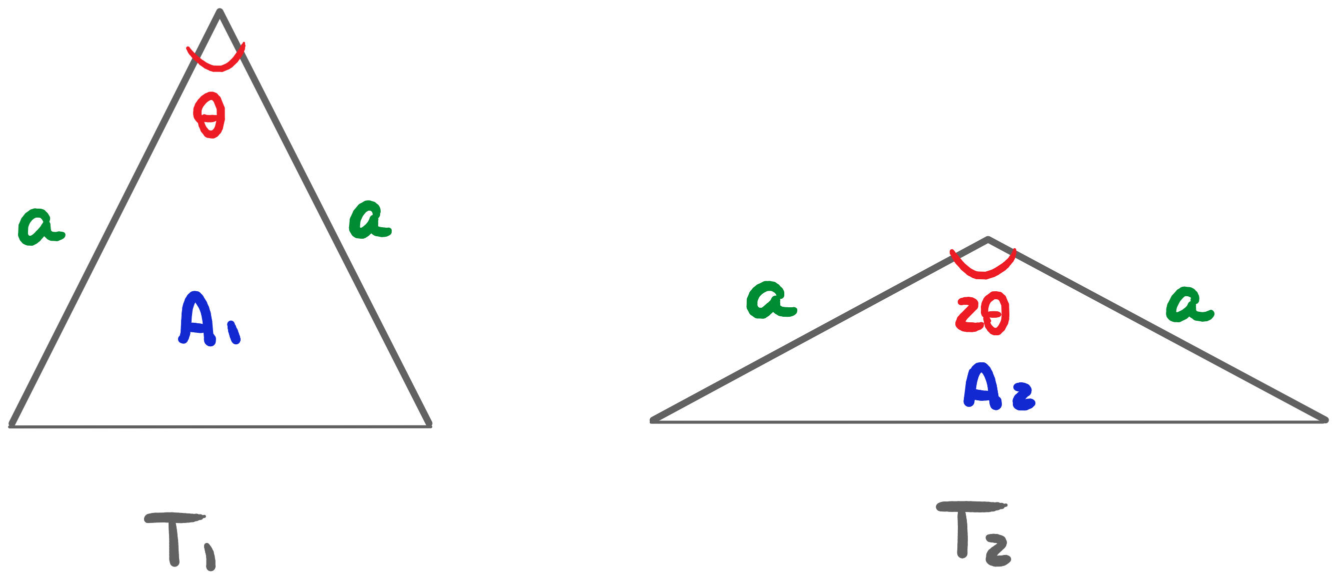
$$\boxed{A_{HEX} = \frac{3a^2\sqrt{3}}{2}}$$

EXEMPLO

MOVENDO AS HASTES DE UM COMPASSO, PODE-SE FORMAR DIVERSOS TRIÂNGULOS, COMO OS DA FIGURA ABAIXO.

SE A ÁREA DO TRIÂNGULO T_1 É O TRIPLO DA ÁREA DO TRIÂNGULO T_2, CALCULE $\cos\theta$.





$$A_1 = \frac{1}{2} \cdot a \cdot a \cdot \text{sen}\,\theta = \frac{1}{2} a^2 \,\text{sen}\,\theta$$

$$A_2 = \frac{1}{2} \cdot a \cdot a \cdot \text{sen}\,2\theta = \frac{1}{2} a^2 \cdot \text{sen}\,2\theta$$

$$A_1 = 3 \cdot A_2$$

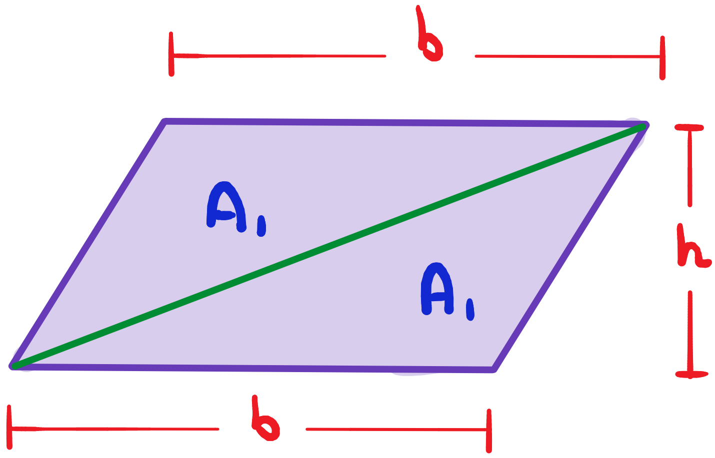
$$\cancel{\frac{1}{2}}\,\cancel{a^2}\,\text{sen}\,\theta = 3\,\cancel{\frac{1}{2}}\,\cancel{a^2} \cdot \overbrace{\text{sen}\,2\theta}^{2 \cdot \text{sen}\,\theta \cdot \cos\theta}$$

$$\cancel{\text{sen}\,\theta} = 3 \cdot 2 \cdot \cancel{\text{sen}\,\theta} \cdot \cos\theta$$

$$1 = 6 \cos\theta$$

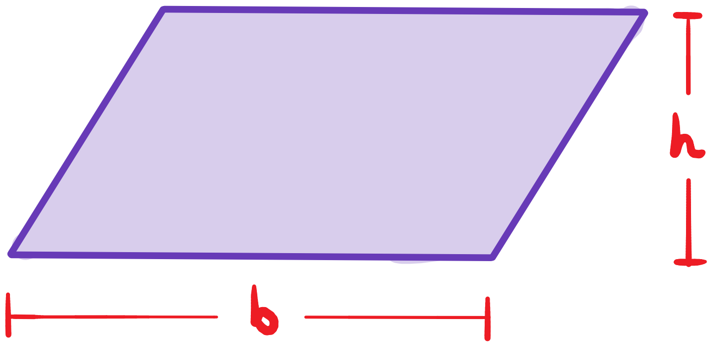
$$\boxed{\cos\theta = \frac{1}{6}}$$

ÁREA DO PARALELOGRAMO

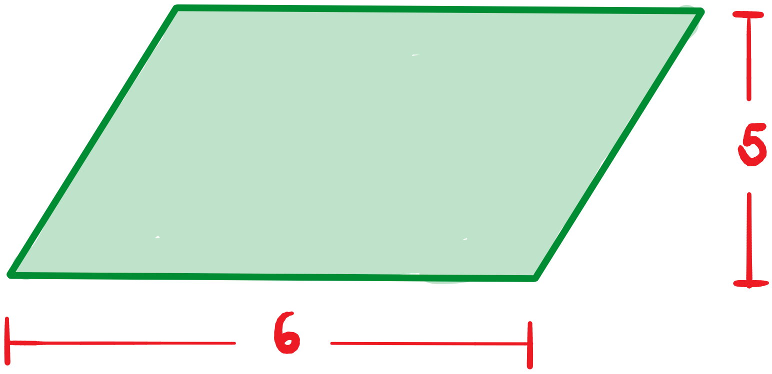


$$A_T = \frac{1}{2} \cdot b \cdot h + \frac{1}{2} \cdot b \cdot h$$

$$A_T = b \cdot h$$



$$\boxed{A = b \cdot h}$$



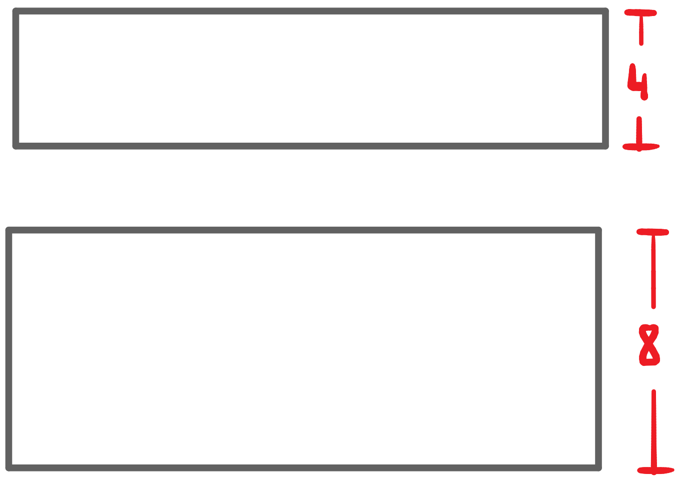
$A = 6.5$

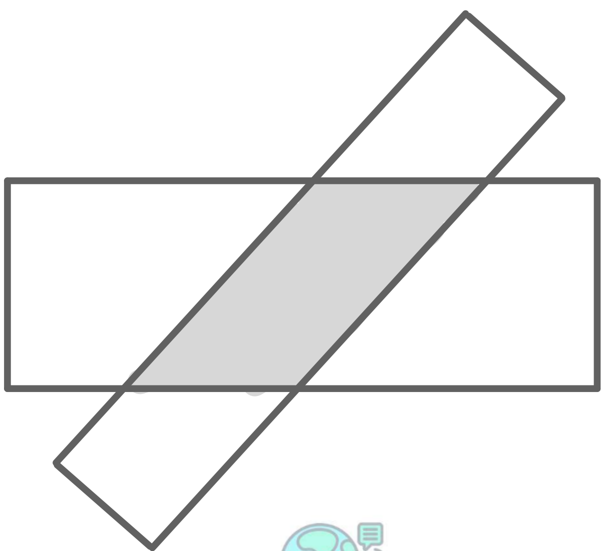
$A = 30$

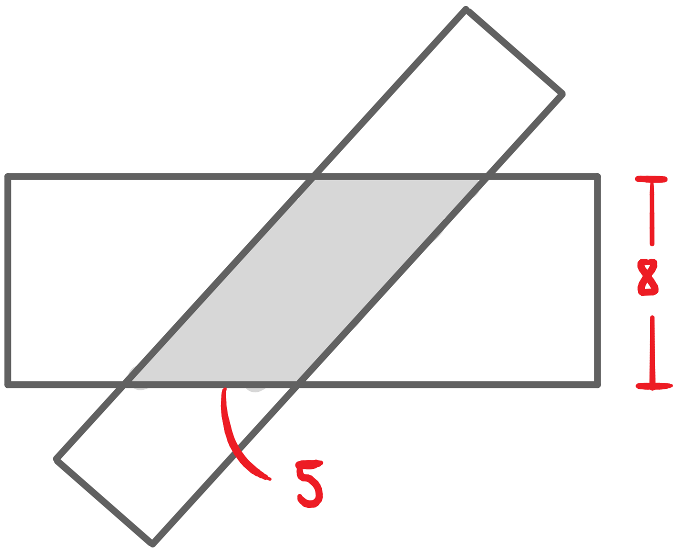
EXEMPLO

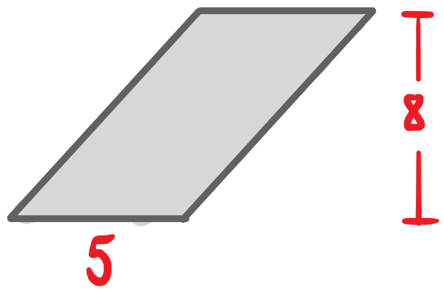
OS DOIS RETÂNGULOS ABAIXO FORAM JUSTAPOSTOS DE FORMA QUE A REGIÃO DE INTERSEÇÃO É UM PARALELOGRAMO.

SE UM DOS LADOS DESSE PARALELOGRAMO É IGUAL A 5, DETERMINE A ÁREA DESSE PARALELOGRAMO.







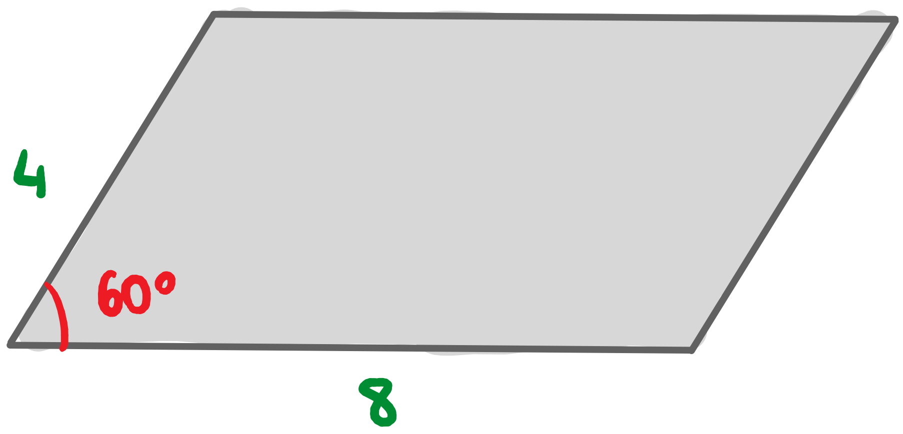


$$A = 5 \cdot 8$$

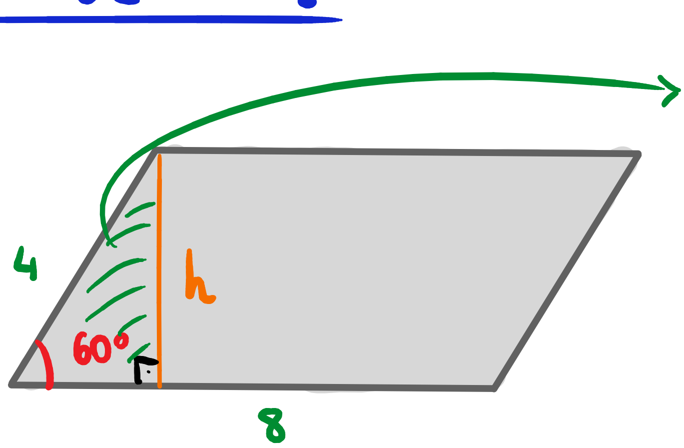
$$A = 40$$

EXEMPLO

CALCULE A ÁREA DO PARALELOGRAMO ABAIXO.



Sol. # 1



$$\sin 60° = \frac{h}{4}$$

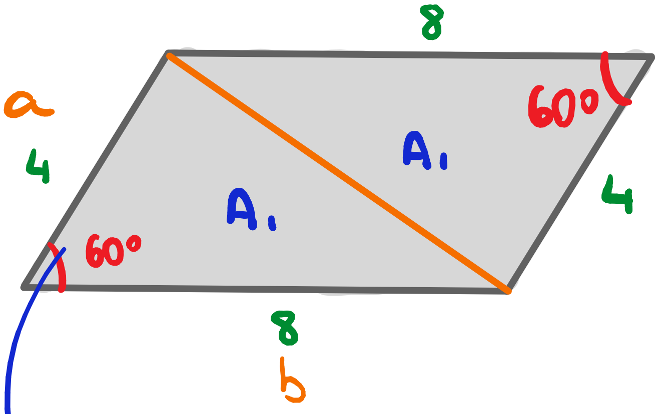
$$\frac{\sqrt{3}}{\cancel{2}} = \frac{h}{\cancel{4}\,2}$$

$$h = 2\sqrt{3}$$

$$A = b.h \rightarrow A = 8.2\sqrt{3}$$

$$\boxed{A = 16\sqrt{3}}$$

Sol. # 2



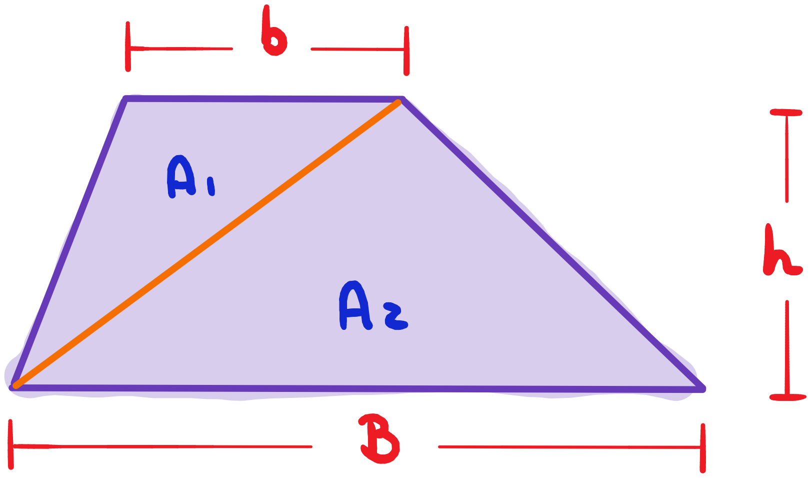
$$A = 2.\frac{1}{2}.a.b.\sin\theta$$

$$A = a.b.\sin\theta$$

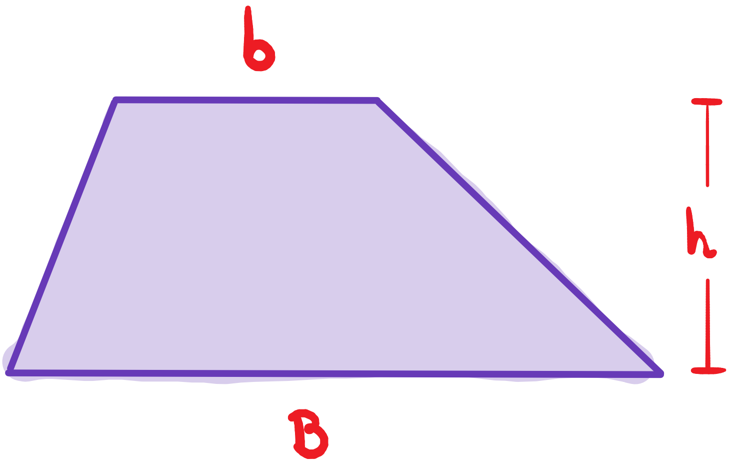
$$A = 4.8.\sin 60° \rightarrow A = 32.\frac{\sqrt{3}}{2}$$

$$\boxed{A = 16\sqrt{3}}$$

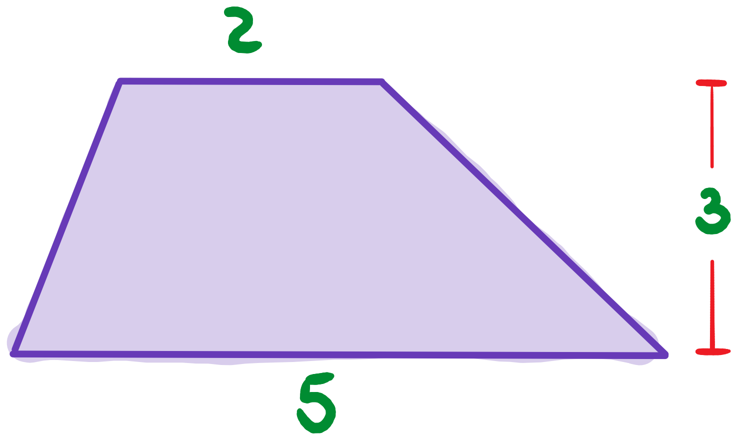
ÁREA DO TRAPÉZIO



$$A = A_1 + A_2 = \frac{b \cdot h}{2} + \frac{B \cdot h}{2} = \frac{(B+b)h}{2}$$



$$A = \frac{(B+b)h}{2}$$



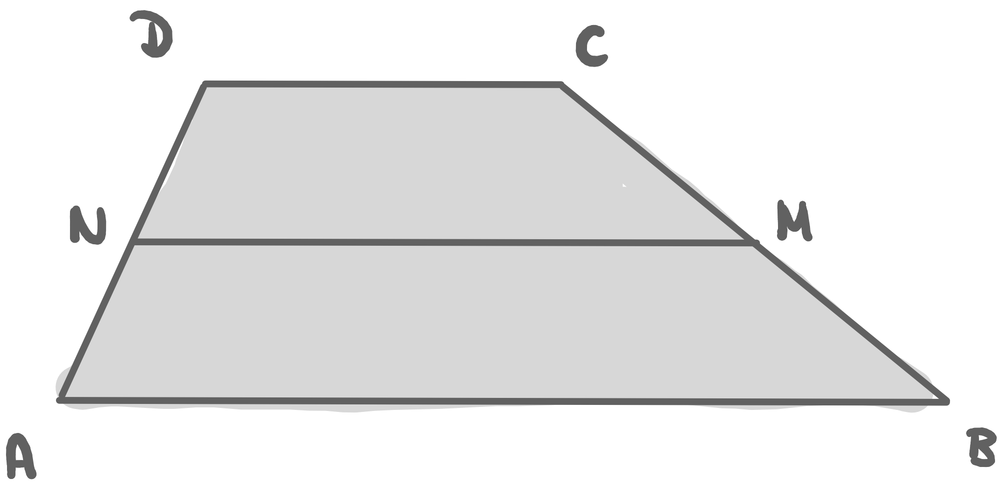
$$A = \frac{(5+2)3}{2}$$

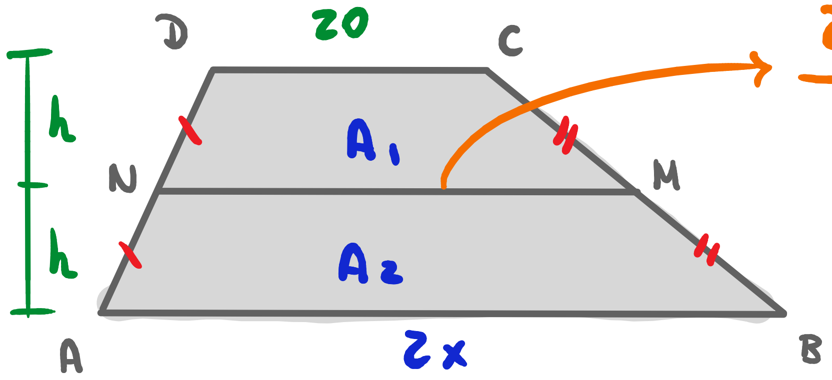
$$\boxed{A = \frac{21}{2}}$$

EXEMPLO

SEJAM M E N OS PONTOS MÉDIOS DOS LADOS BC E AD DO TRAPÉZIO. O SEGMENTO DIVIDE O TRAPÉZIO ABCD EM ÁREAS PROPORCIONAIS A 1 E A 2.

SE CD = 20, CALCULE O COMPRIMENTO DE AB.





$$\frac{2x + 20}{2} = x + 10$$

$$A_1 = \frac{(x + 10 + 20)h}{2} = \frac{(x + 30)h}{2}$$

$$A_2 = \frac{(2x + x + 10)h}{2} = \frac{(3x + 10)h}{2}$$

$$A_2 = 2 . A_1$$

$$\frac{(3x + 10)\cancel{h}}{\cancel{2}} = 2 . \frac{(x + 30)\cancel{h}}{\cancel{2}}$$

$$3x + 10 = 2x + 60$$

$$\underline{x = 50}$$

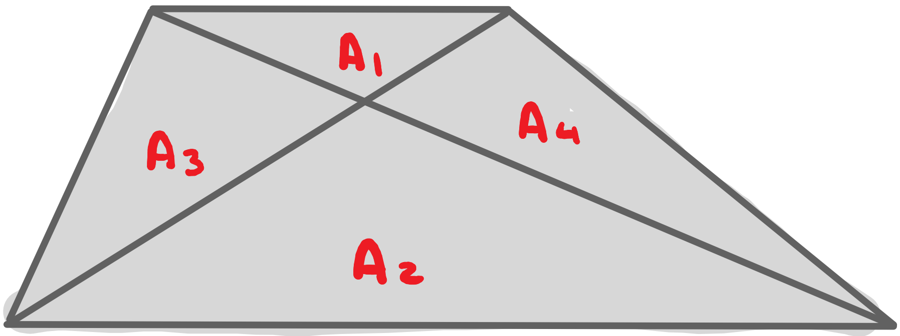
$$AB = 2x \longrightarrow \boxed{AB = 100}$$

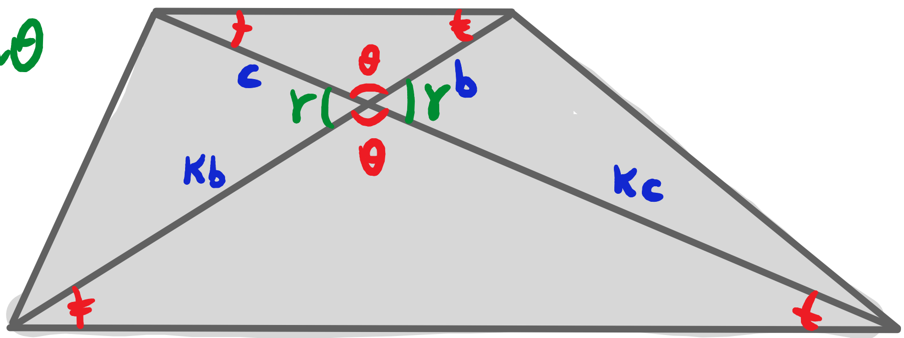
EXEMPLO

SEJA O TRAPÉZIO ABCD, DE BASES AB E CD.

AS DIAGONAIS SE CORTAM NO PONTO I.

CALCULE A ÁREA DO TRIÂNGULO ADI SABENDO QUE AS ÁREAS DOS TRIÂNGULOS ABI E CDI SÃO, RESPECTIVAMENTE 12 E 3.





$$A_1 = \frac{1}{2} . bc \operatorname{sen}\theta \quad ; \quad A_2 = \frac{1}{2} Kb . Kc . \operatorname{sen}\theta$$

$$A_2 = K^2 . \frac{1}{2} bc \operatorname{sen}\theta$$

$$A_3 = \frac{1}{2} c . Kb . \operatorname{sen}\gamma \rightarrow A_3 = K . \frac{1}{2} bc \operatorname{sen}\theta$$

$$A_4 = \frac{1}{2} . b . Kc . \operatorname{sen}\gamma \rightarrow A_4 = K . \frac{1}{2} bc . \operatorname{sen}\theta$$

$$A_3 = A_4$$

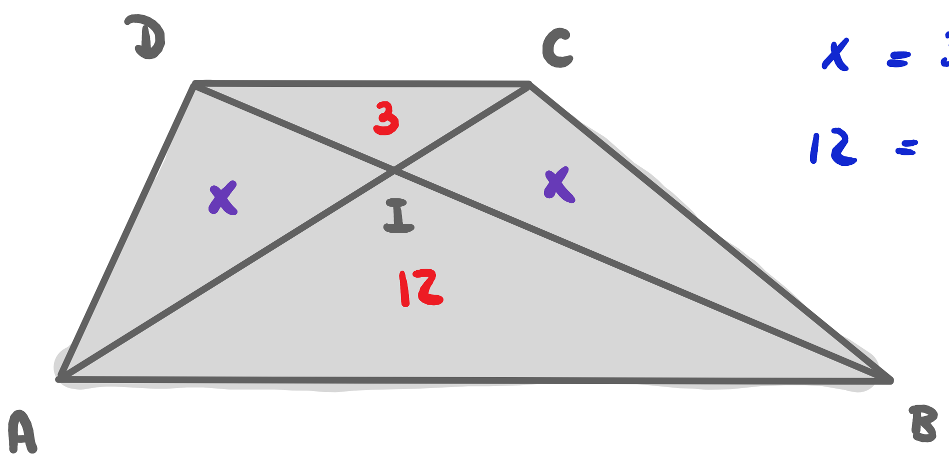
$$A_1 = \frac{1}{2} \cdot bc\,\text{sen}\,\theta \quad \cdot K$$

$$A_3 = A_4 = K \cdot \frac{1}{2}\, bc\,\text{sen}\,\theta \quad \cdot K$$

$$A_2 = K^2 \cdot \frac{1}{2}\, bc\,\text{sen}\,\theta$$

PROG. GEOMÉTRICA $(A_1,\ A_3,\ A_2)$

$$A_3 = \sqrt{A_1 \cdot A_2}$$



$$x = 3 \cdot K \rightarrow K = \frac{x}{3}$$

$$12 = x \cdot K \rightarrow K = \frac{12}{x}$$

$$\frac{x}{3} = \frac{12}{x}$$

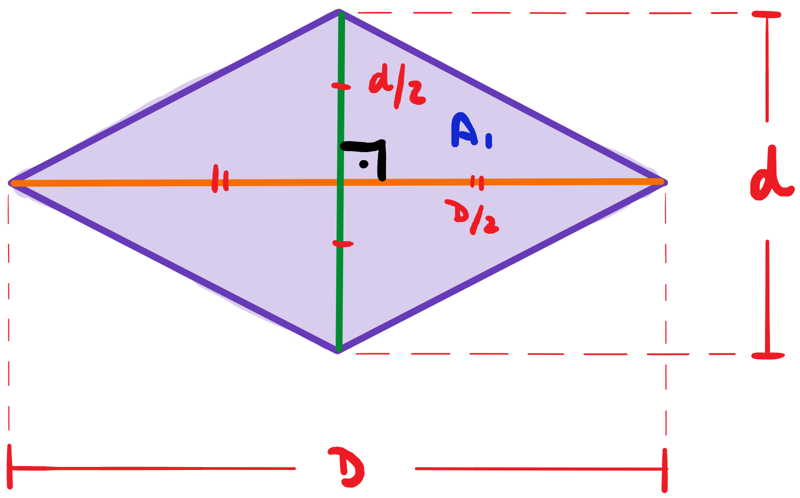
$$x^2 = 36$$

$$x = 6$$

$$x = \sqrt{3 \cdot 12} \rightarrow x = \sqrt{36}$$

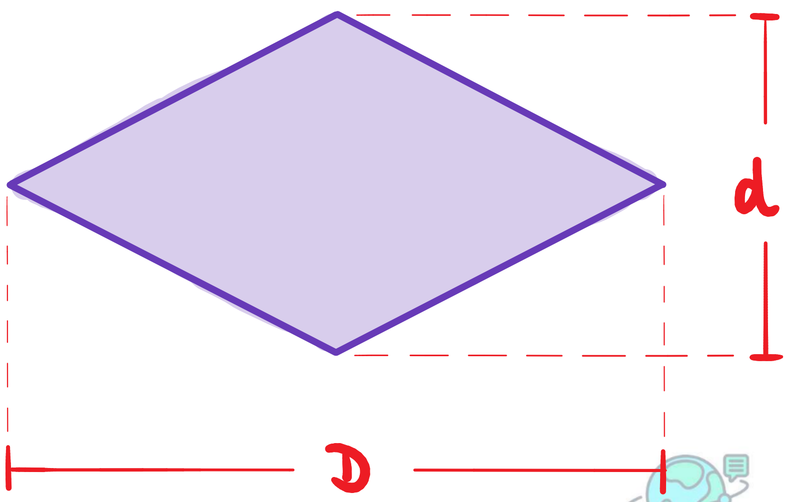
$$x = 6$$

ÁREA DO LOSANGO

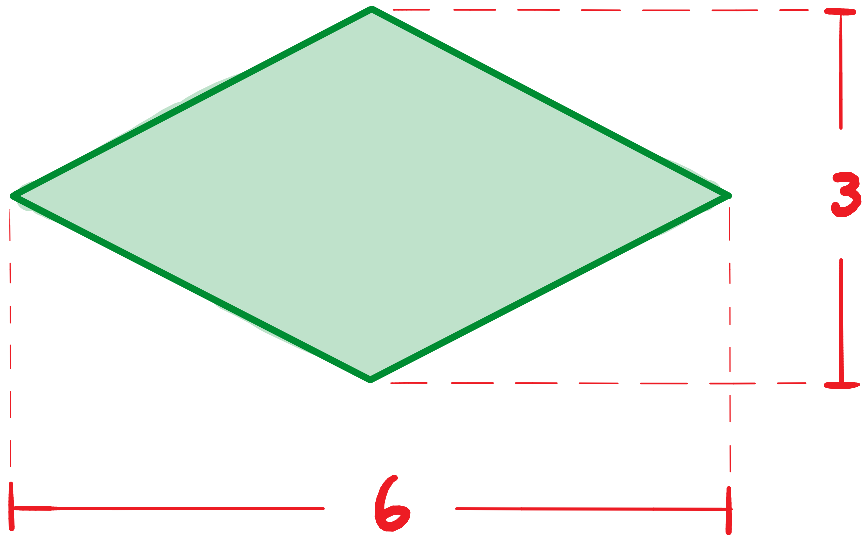


$$A_L = 4 \cdot A_1$$

$$A_L = \cancel{4} \cdot \frac{1}{\cancel{2}} \cdot \frac{D}{\cancel{2}} \cdot \frac{d}{2} \rightarrow A_L = \frac{D \cdot d}{2}$$



$$\boxed{A = \frac{D \cdot d}{2}}$$



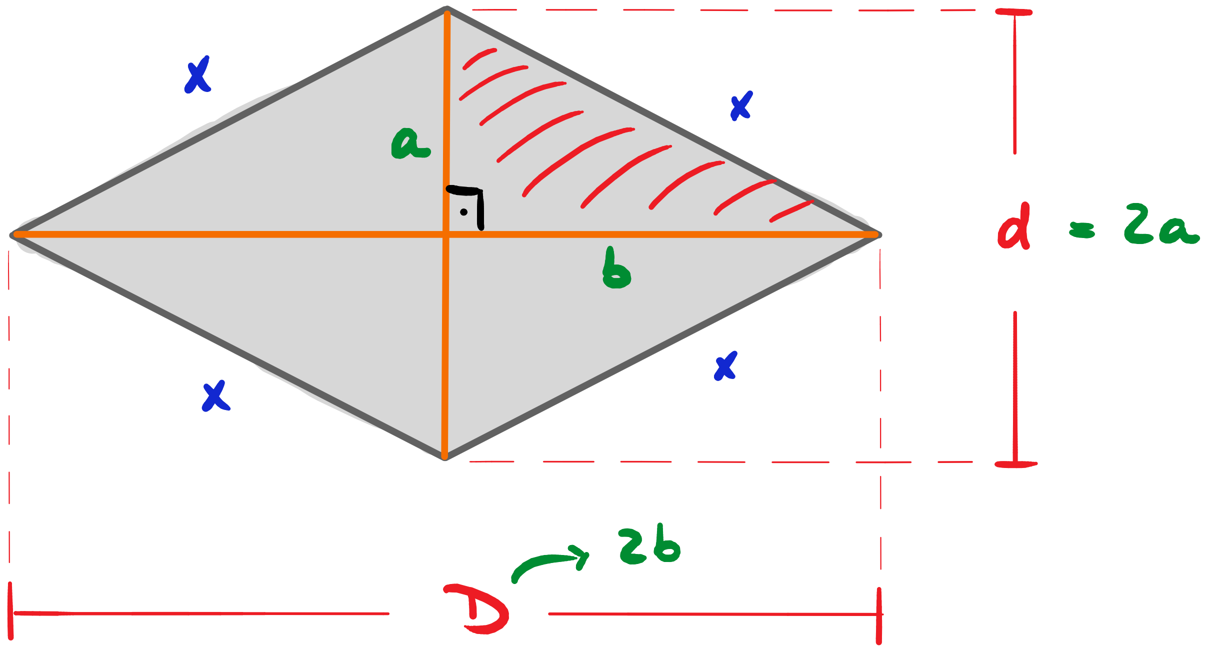
$$A = \frac{1}{\cancel{2}} \cdot \overset{3}{\cancel{6}} \cdot 3$$

$$A = 9$$

EXEMPLO

A ÁREA E O PERÍMETRO DE UM LOSANGO SÃO, RESPECTIVAMENTE, $24m^2$ E 20m.

CALCULE OS VALORES DAS DIAGONAIS DESSE LOSANGO.



$$4x = 20 \rightarrow \underline{x = 5}$$

$$\overset{12}{\cancel{24}} = \frac{\cancel{2}b \cdot \cancel{2}a}{\cancel{2}} \longrightarrow \underline{a \cdot b = 12}$$

$$a^2 + b^2 = 5^2 \rightarrow \underline{a^2 + b^2 = 25}$$

$$(a+b)^2 = \underbrace{a^2 + b^2}_{25} + 2\underbrace{ab}_{12}$$

$$(a+b)^2 = 49$$

$$\underline{a + b = 7}$$

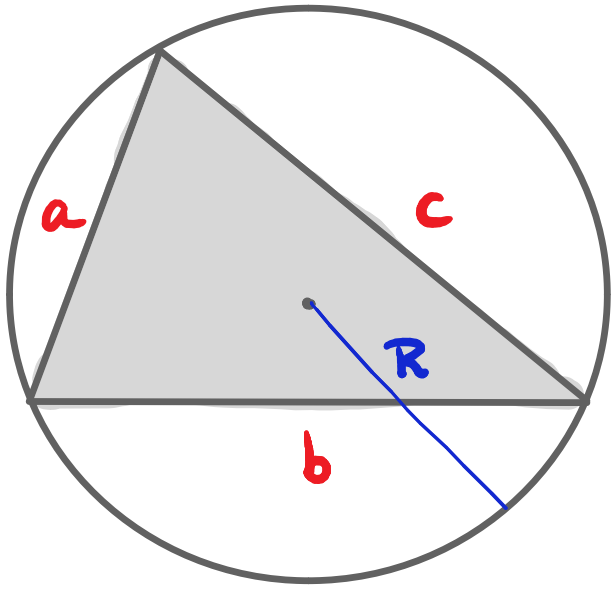
$$\boxed{\begin{matrix} a + b = 7 \\ a \cdot b = 12 \end{matrix}} \rightarrow \begin{matrix} a = 3 \\ b = 4 \end{matrix}$$

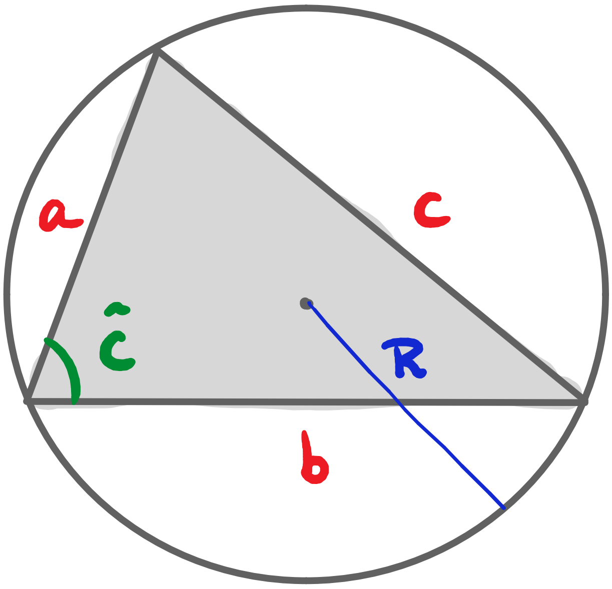
$$D = 8 \; ; \; d = 6$$

EXEMPLO

MOSTRE QUE A ÁREA DO TRIÂNGULO PODE SER ESCRITA COMO:

$$A = \frac{a.b.c}{4R}$$





$$A = \frac{1}{2} \cdot a \cdot b \cdot \operatorname{sen} \hat{C}$$

LEI DOS SENOS:

$$\frac{c}{\operatorname{sen} \hat{C}} = 2R \rightarrow \operatorname{sen} \hat{C} = \frac{c}{2R}$$

$$A = \frac{1}{2} \cdot a \cdot b \cdot \frac{c}{2R}$$

$$\boxed{A = \frac{a\,b\,c}{4R}}$$

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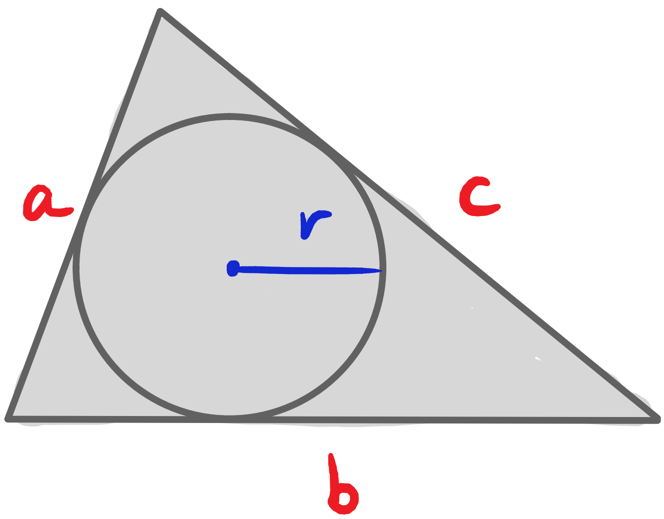
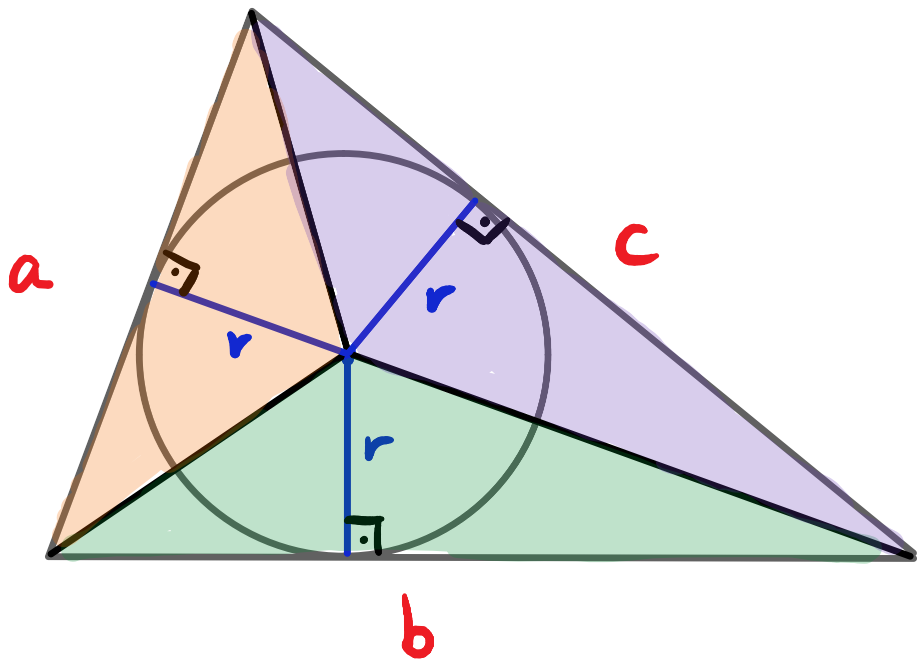
EXEMPLO

SENDO p O SEMI-PERÍMETRO DE UM TRIÂNGULO E r O RAIO DA CIRCUNFERÊNCIA INSCRITA A ELE, MOSTRE QUE SUA ÁREA É DADA POR:

$$A = p.r$$





$$A_\Delta = A_1 + A_2 + A_3$$

$$= \frac{1}{2}.ar + \frac{1}{2}b.r + \frac{1}{2}.c.r$$

$$= r\left(\frac{a}{2} + \frac{b}{2} + \frac{c}{2}\right)$$

$$= r\left(\frac{a + b + c}{2}\right)$$

$$A_\Delta = p.r$$

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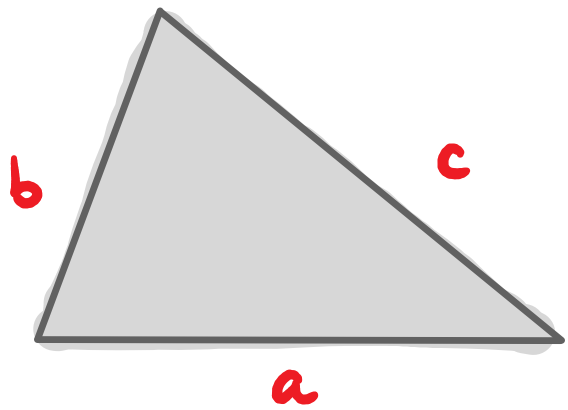
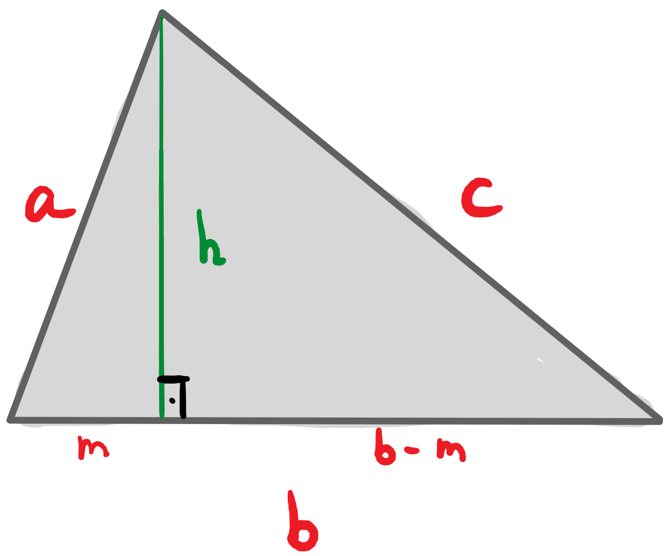
EXEMPLO

CONSIDERE UM TRIÂNGULO QUALQUER COMO O DA FIGURA ABAIXO.



SENDO p SEU SEMI-PERÍMETRO, DEMONSTRE A FÓRMULA DE HERON PARA O CÁLCULO DA ÁREA.

$$A = \sqrt{p(p-a)(p-b)(p-c)}$$



$$P = \frac{a + b + c}{2}$$

$$A = \frac{1}{2} bh$$

$$\begin{cases} a^2 = h^2 + m^2 \\ c^2 = h^2 + b^2 - 2bm + m^2 \end{cases}$$

$$-$$

$$a^2 - c^2 = -b^2 + 2bm$$

$$m = \frac{a^2 + b^2 - c^2}{2b}$$

$$a^2 = h^2 + \left(\frac{a^2 + b^2 - c^2}{2b} \right)^2$$

$$\frac{4a^2b^2 = 4b^2h^2 + \left(a^2+b^2-c^2\right)^2}{\cancel{4b^2}}$$

$$4b^2h^2 = 4a^2b^2 - \left(a^2+b^2-c^2\right)^2$$

$$16\left(\frac{1}{2}bh\right)^2 = (2ab)^2 - \left(a^2+b^2-c^2\right)^2$$

$$16.A^2 = \left(2ab+a^2+b^2-c^2\right)\left(2ab-a^2-b^2+c^2\right)$$

$$16A^2 = \left(a^2+2ab+b^2-c^2\right)\left(c^2-\left(a^2-2ab+b^2\right)\right)$$

$$16A^2 = \left[(a+b)^2-c^2\right]\left[c^2-(a-b)^2\right]$$

$$16A^2 = (a+b+c)(a+b-c)(c+a-b)(c-a+b)$$

$$16A^2 = (a+b+c)(a+b-c)(a-b+c)(-a+b+c)$$

$$p = \frac{a+b+c}{2} \rightarrow a+b+c = 2p$$

$$a+b+c-2c = 2p-2c$$

$$a+b-c = 2(p-c)$$

$$a-b+c = 2(p-b)$$

$$-a+b+c = 2(p-a)$$

$$1\cancel{6}A^2 = \cancel{2}p \cdot \cancel{2}(p-c) \cdot \cancel{2}(p-b) \cdot \cancel{2}(p-a)$$

$$\boxed{A = \sqrt{p(p-a)(p-b)(p-c)}}$$

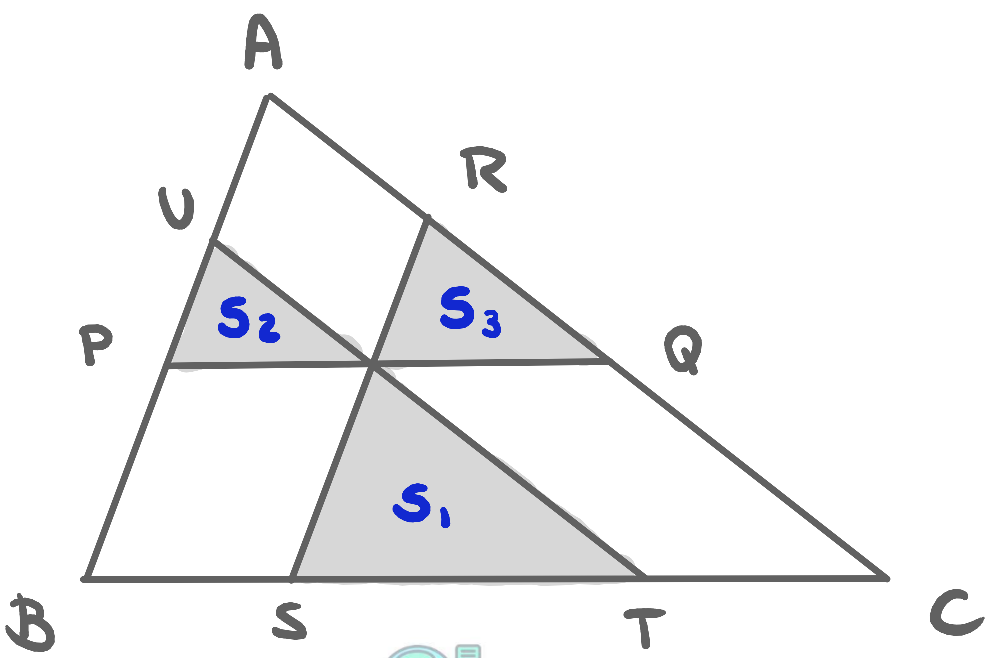
EXEMPLO

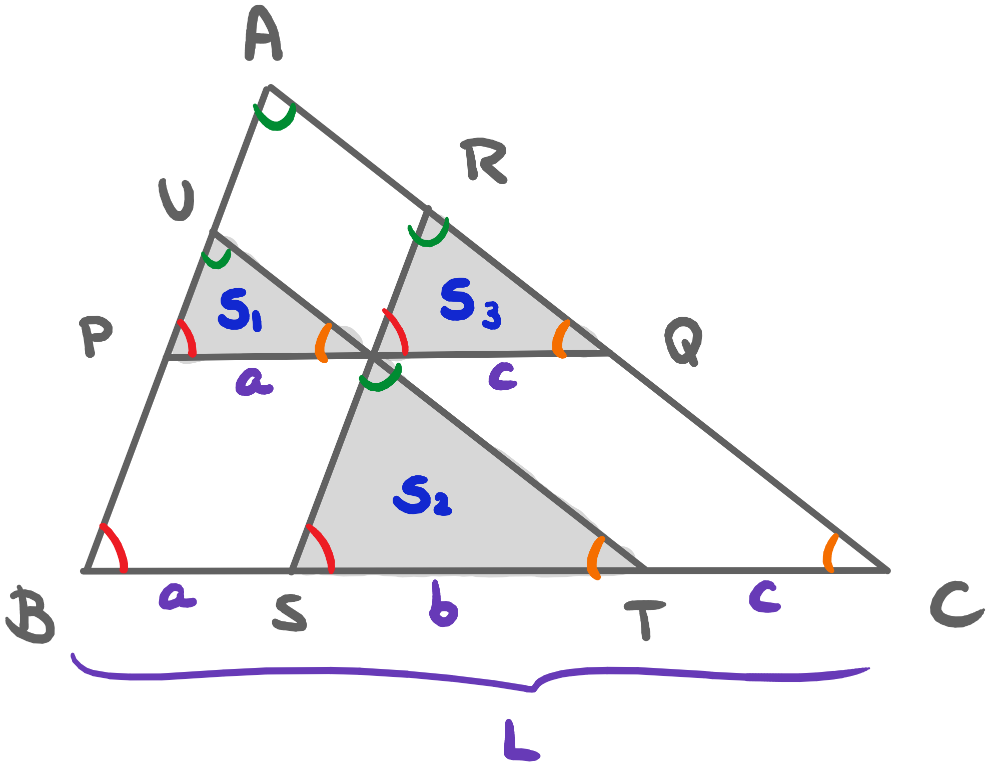
SEJA O TRIÂNGULO ABC MOSTRADO NA FIGURA, TAL QUE:

ÁREA DO TRIÂNGULO ABC: S
PQ // BC, RS // AB, TU // AC.
PQ, RS E TU SE INTERSEPTAM EM P.

SENDO S_1, S_2 E S_3 AS ÁREAS DESTACADAS, MOSTRE QUE:

$$\sqrt{S} = \sqrt{S_1} + \sqrt{S_2} + \sqrt{S_3}$$





$$K_1 = \frac{a}{L} \; ; \quad \frac{S_1}{S} = K_1^2 \;\rightarrow\; \frac{S_1}{S} = \left(\frac{a}{L}\right)^2$$

$$K_2 = \frac{b}{L} \; ; \quad \frac{S_2}{S} = K_2^2 \;\rightarrow\; \frac{S_2}{S} = \left(\frac{b}{L}\right)^2$$

$$K_3 = \frac{c}{L} \; ; \quad \frac{S_3}{S} = K_3 \;\rightarrow\; \frac{S_3}{S} = \left(\frac{c}{L}\right)^2$$

$$\frac{a}{L} = \sqrt{\frac{S_1}{S}} \; ; \quad \frac{b}{L} = \sqrt{\frac{S_2}{S}} \; ; \quad \frac{c}{L} = \sqrt{\frac{S_3}{S}}$$

$$K_1 + K_2 + K_3 = 1$$

$$\frac{a}{L} + \frac{b}{L} + \frac{c}{L} = 1$$

$$\frac{a + b + c}{L} = \frac{L}{L} = 1$$

$$\frac{a}{L} + \frac{b}{L} + \frac{c}{L} = 1$$

$$\sqrt{\frac{S_1}{S}} + \sqrt{\frac{S_2}{S}} + \sqrt{\frac{S_3}{S}} = 1$$

$$\cancel{\sqrt{S}}\left(\frac{\sqrt{S_1}}{\cancel{\sqrt{S}}} + \frac{\sqrt{S_2}}{\cancel{\sqrt{S}}} + \frac{\sqrt{S_3}}{\cancel{\sqrt{S}}}\right) = 1 \cdot \sqrt{S}$$

$$\boxed{\sqrt{S_1} + \sqrt{S_2} + \sqrt{S_3} = \sqrt{S}}$$

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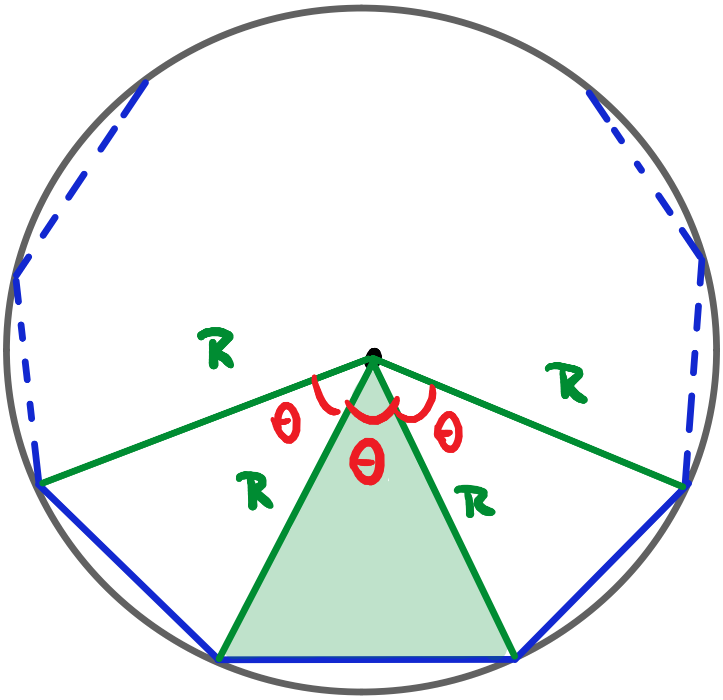
EXEMPLO

SEJA UM POLÍGONO REGULAR DE n LADOS INSCRITO EM UMA CIRCUNFERÊNCIA DE RAIO R.

CALCULE A ÁREA DESSE POLÍGONO.



$$n \cdot \theta = 360^\circ$$

$$\theta = \frac{360^\circ}{n}$$

$$A_1 = \frac{1}{2} \cdot R \cdot R \cdot \operatorname{sen} \theta$$

$$A_1 = \frac{1}{2} \cdot R^2 \cdot \operatorname{sen}\left(\frac{360^\circ}{n}\right)$$

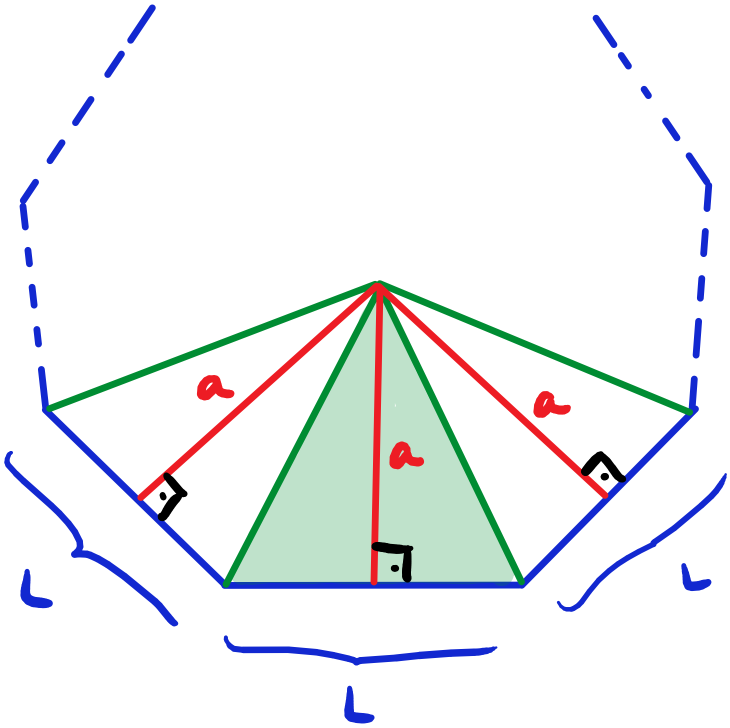
$$A_{TOTAL} = n \cdot A_1$$

$$\boxed{A_{TOTAL} = n \cdot \frac{1}{2} \cdot R^2 \cdot \operatorname{sen}\left(\frac{360^\circ}{n}\right)}$$

EXEMPLO

SEJA UM POLÍGONO REGULAR DE PERÍMETRO $2p$ E APÓTEMA a.

CALCULE A ÁREA DESSE POLÍGONO.



$$A_1 = \frac{1}{2} \cdot L \cdot a$$

$$2p = nL \rightarrow L = \frac{2p}{n}$$

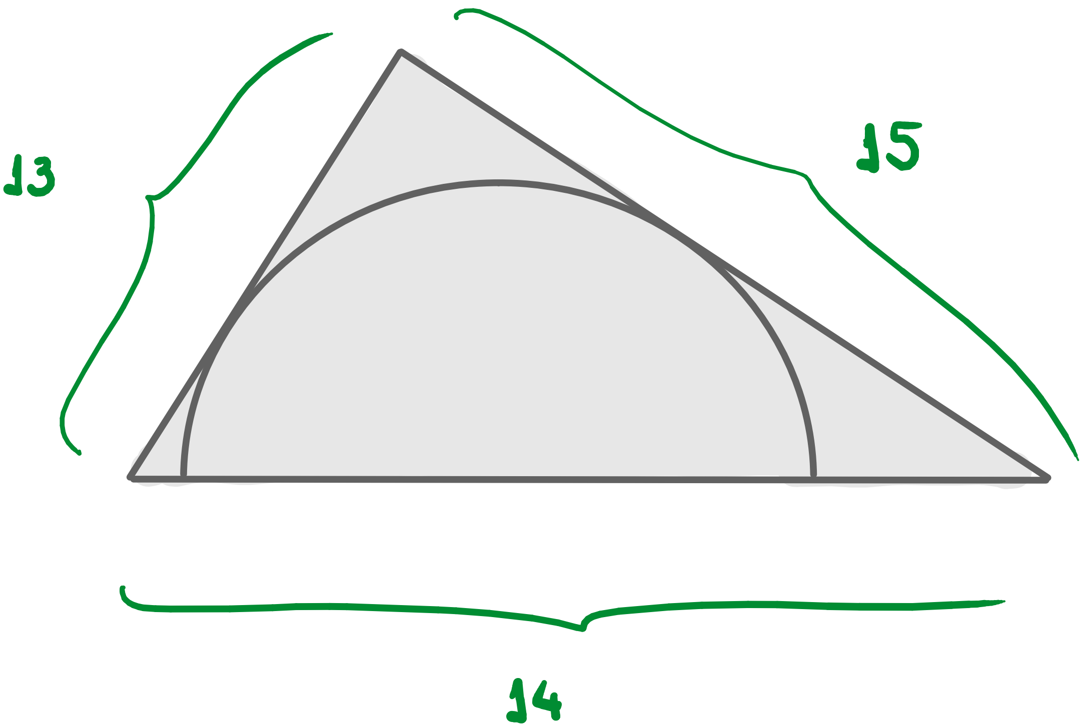
$$A_1 = \frac{1}{\cancel{2}} a \cdot \frac{\cancel{2}p}{n} \rightarrow A_1 = \frac{ap}{n}$$

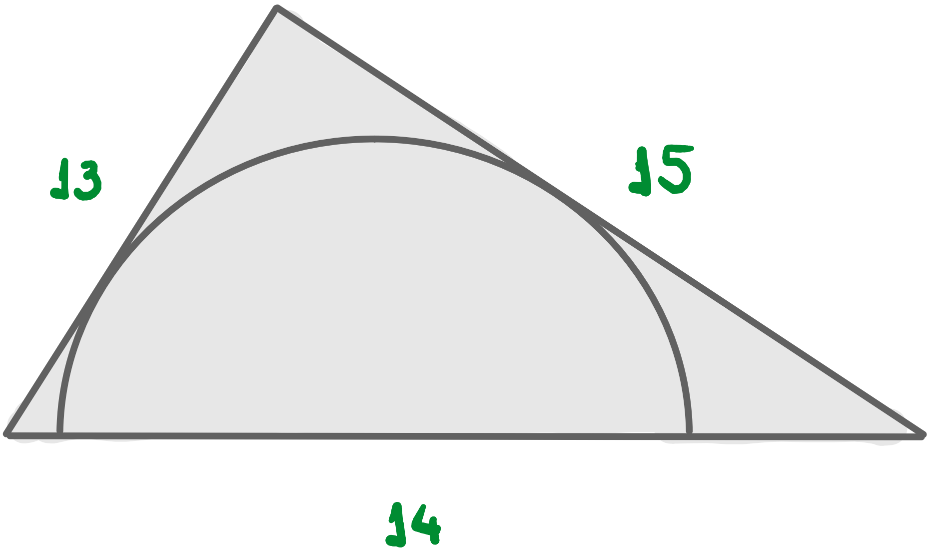
$$A_T = n \cdot A_1 \longrightarrow A_T = \cancel{n} \cdot \frac{ap}{\cancel{n}}$$

$$\boxed{A_T = a \cdot p}$$

EXEMPLO

DETERMINE O RAIO DA SEMI-CIRCUNFERÊNCIA INSCRITA AO TRIÂNGULO ABAIXO.





$$A = \sqrt{p(p-a)(p-b)(p-c)}$$

$$p = \frac{13 + 14 + 15}{2} = 21$$

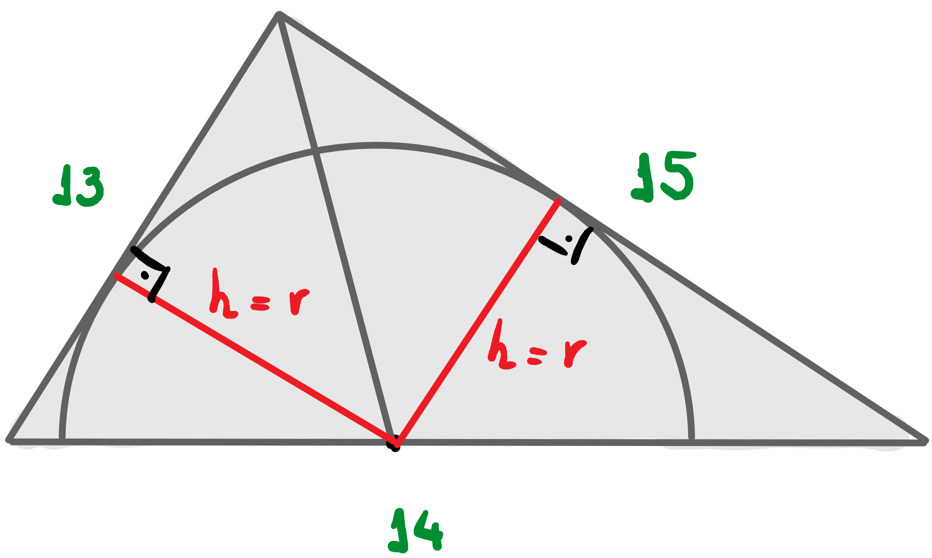
$$A_{\Delta} = \sqrt{21(21-13)(21-14)(21-15)}$$

$$A_{\Delta} = \sqrt{21 . 8 . 7 . 6}$$

$$A_{\Delta} = \sqrt{\cancel{3}.\cancel{7}.\cancel{2}.\cancel{2}.\cancel{2}.\cancel{7}.\cancel{2}.\cancel{3}}$$

$$A_{\Delta} = 7 . 2 . 2 . 3$$

$$A_{\Delta} = 84$$



$$A_\Delta = \frac{1}{2} \cdot 13r + \frac{1}{2} \cdot 15r$$

$$A_\Delta = \frac{28r}{2}$$

$$A_\Delta = 14r$$

$$A_\Delta = A_\Delta$$

$$14r = 84$$

$$r = 6$$